<u>Upward Bound Summer 2015</u>

Pre-Calculus Curriculum

Instructor: Sean Cabaniss **Email:** sean.cabaniss@umfupwardbound.org

Study sessions located in: Learning Commons--Mantor Library

<u>Week 1</u>

- Day 1: Get to know/ Pre Test
- Day 2: Completing the square
- Day 3: Completing the square to graph roots and quadratics

<u>Week 2</u>

- Day 1: Composition and decomposition of functions
- Day 2: Inverse functions
- Day 3: Basic Graphs & Transformations

<u>Week 3</u>

- Day 1: Vertical asymptotes and removable discontinuities
- Day 2: Horizontal asymptotes, slant asymptotes, and end behavior
- Day 3: Graphing all kinds of different functions

<u>Week 4</u>

- Day 1: Introduce unit circle and all of its glory, finding points around circle
- Day 2: Create graphs of sin(x), cos(x), tan(x) by examining points
- Day 3: Getting more comfortable with the graphs and their points

<u>Week 5</u>

- Day 1: Transformations of sin and cos graphs
- Day 2: Roller Coaster project
- Day 3: Start gathering resources/begin review

<u>Week 6</u>

- Day 1 – Review / Activity (depending on which classes won't be here Friday)
- Day 2 - Post Test/ Review (depending on which classes won't be here Friday)
- Day 3 – Post Test

<u>Missing/ Incomplete Homework</u>:

- 1 missed homework = study out of free time until it is finished
- 3 study outs = 1 Class C violation
- 2 Class C's for homework will result in a Disciplinary Meeting

Pre-Calculus Pre-Test

Name:____________________________ Date:________________

Simplify each expression

1. $-7(n+3)-8(1-8n)$

2. $x^3+2x-4x^4+2x^3-4x-1$

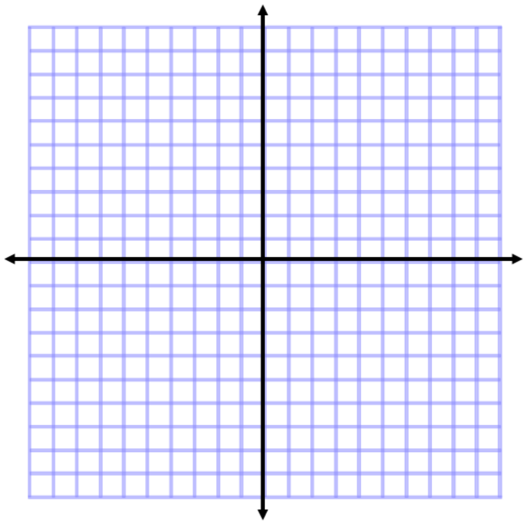
Solve each expression

3. $26=8+v$

4. $-18-6k=6(1+3k)$

For questions 6 through 8, refer to the equation, $y = 2x - 5$.

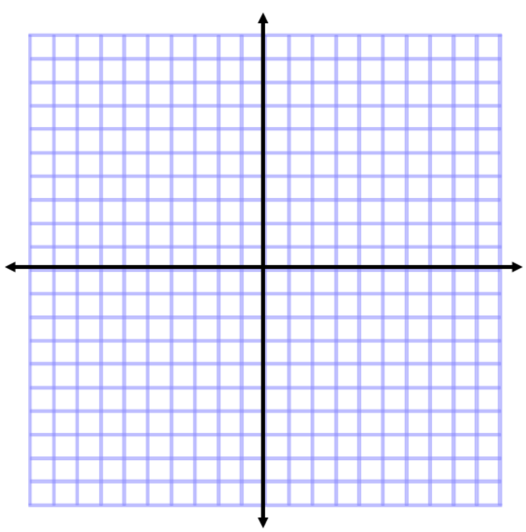
5. Graph the equation



6. What is the slope of the line?

7. What is the y-intercept of the line?

For questions 8 and 9, refer to the equation: $f(x) = 2(x+3)^2 - 1$

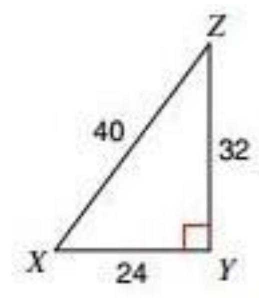


8. Graph the equation

9. What is the vertex?

10. Refer to figure 1: What is $\tan(X)$?

Figure 1



For questions 11 and 12 refer to the following equations:

$$f(x) = 3x^2 + 6$$
$$g(x) = 9 - x$$

11. Find $f(2)$

12. Find $g(f(x))$

13. Is this function continuous or discontinuous? Why? $\dfrac{1}{2x+1}$

Convert the following from degrees to radians

14. 90˚

15. 12˚

16. Rewrite the following in exponential form: $\log_6 36 = 2$

Week 1!

Class 1:

- Introductions (5 min)
- Gifts (5 min)
- Syllabus (5 min)
- Pre-Test (20 min)
- Questions (5 min)
- Talk (yeah)

Class 2:

- SAT QOD (5 min)
- Vid (5 min)
- Review pre-test (10 min)
- Review factoring (15 min)
  - Use factoring flip cards
- Show completing the square (20 min)
  - Know that a perfect square is in the form $(x+a)^2$ which equals (x+a)(x+a). Then set the two equations equal to each other and match up based on position.
    - x^2+6x+c where c will equal 9
    - $r^2+20r+c$ where c will equal 100
    - $x^2-34x+c$ where c will equal 289
  - Coaching activity with a couple of problems
    - $x^2-11x+c$
    - x^2+4x+c
    - $q^2+17q+c$
    - $p^2+24p+c$
- Leave them with video (5 min)
- MATH MISSION:
  - Cross off 1, 6, and 3, we did those
  - 2 thru 8, 13, 17 are worth 1 point, 10, 12, 14, 15 are worth 3 points, the rest are worth 5 points. You need to do at least one from each section, and a total of five problems. Most points next class wins!
  - http://cdn.kutasoftware.com/Worksheets/Alg2/Completing%20the%20Square.pdf

Class 3: LOTS OF STUFF GET A MOVE ON!

- SAT QOD (5 min)
- Go over homework questions (10 min)
- Look at solving to complete the square (25 min)
    - Show them trick with taking half of b and squaring it to get the correct value for c.
    - Should work for all examples
    - Show completing the square with diagram. DON'T FORGET TO TAKE THE PLUS OR MINUS AFTER TAKING SQUARE ROOT.
        - $x^2+14x-51=0$, should be $x=3,-17$
        - $x^2+6x+8=0$, should be $x=-2,-4$
        - $x^2-12x+11=0$, should be $x=11,1$
    - Go over graphing, domain and range.
        - This one will show up on your homework, write it down!
            - $y=3x^2+30x+74$, need to get it to be $-74+3(25)=3(x^2+10x+25)$, then to $1=3(x+5)^2$
        - SHOW COMPLETING THE SQUARE TO GET QUADRATIC FORMULA (If there's time)
            - Start with $ax^2+bx+c=0$, then divide both sides by a to get the leading coefficient equal to 1. Complete the square, should get $(x+b/2a)^2$
- Homework answers: roots = -5,-1, vertex at (-3,-4) domain=all real numbers, range is y is greater than or equal to -4. Second problem: roots=2+sqrt(13), 2-sqrt(13), vertex at (2,-26), domain=all reals, range is greater than or equal to -26
- Perhaps leave with video if there's time (5 min)

If Time:

Introduce polynomial expansion: $(x+y)^n$ *with binomial theorem*

- *Mission is to do two problems with binomial theorem expansion*
- *Find video for help if needed. Make one if needed.*

- *Introduce Pascal's Triangle!!! GLORIOUS (5 min)*
- *Cut up pieces of paper with partners to have them put together the triangle. Down to the* 7^{th} *row!!! Booyah! (10 min)*

Do a couple of problems with expanding pascal's triangle. USE EGG POINTERS IN GROUPS OF TWO!

MATH BINGO

Kuta Software - Infinite Algebra 2

Name____________________________________

Completing the Square

Date________________ Period____

Find the value of *c* that completes the square.

1) $x^2 + 6x + c$

2) $z^2 - 10z + c$

3) $x^2 - 34x + c$

4) $r^2 + 32r + c$

5) $r^2 - 6r + c$

6) $r^2 + 20r + c$

7) $x^2 - 38x + c$

8) $a^2 + 12a + c$

9) $x^2 - \frac{25}{13}x + c$

10) $a^2 - 7a + c$

11) $z^2 + \frac{11}{8}z + c$

12) $m^2 + 3m + c$

13) $m^2 + 40m + c$

14) $x^2 + 13x + c$

15) $x^2 - x + c$

16) $n^2 - \frac{1}{2}n + c$

17) $a^2 - 8a + c$

18) $x^2 + \frac{7}{13}x + c$

Kuta Software - Infinite Algebra 2 Name________________________________

Completing the Square

Date________________ Period____

Find the value of *c* that completes the square.

1) $x^2 + 6x + c$

9

2) $z^2 - 10z + c$

25

3) $x^2 - 34x + c$

289

4) $r^2 + 32r + c$

256

5) $r^2 - 6r + c$

9

6) $r^2 + 20r + c$

100

7) $x^2 - 38x + c$

361

8) $a^2 + 12a + c$

36

9) $x^2 - \frac{25}{13}x + c$

$\frac{625}{676}$

10) $a^2 - 7a + c$

$\frac{49}{4}$

11) $z^2 + \frac{11}{8}z + c$

$\frac{121}{256}$

12) $m^2 + 3m + c$

$\frac{9}{4}$

13) $m^2 + 40m + c$

400

14) $x^2 + 13x + c$

$\frac{169}{4}$

15) $x^2 - x + c$

$\frac{1}{4}$

16) $n^2 - \frac{1}{2}n + c$ $\frac{1}{16}$

17) $a^2 - 8a + c$ 16

18) $x^2 + \frac{7}{13}x + c$ $\frac{49}{676}$

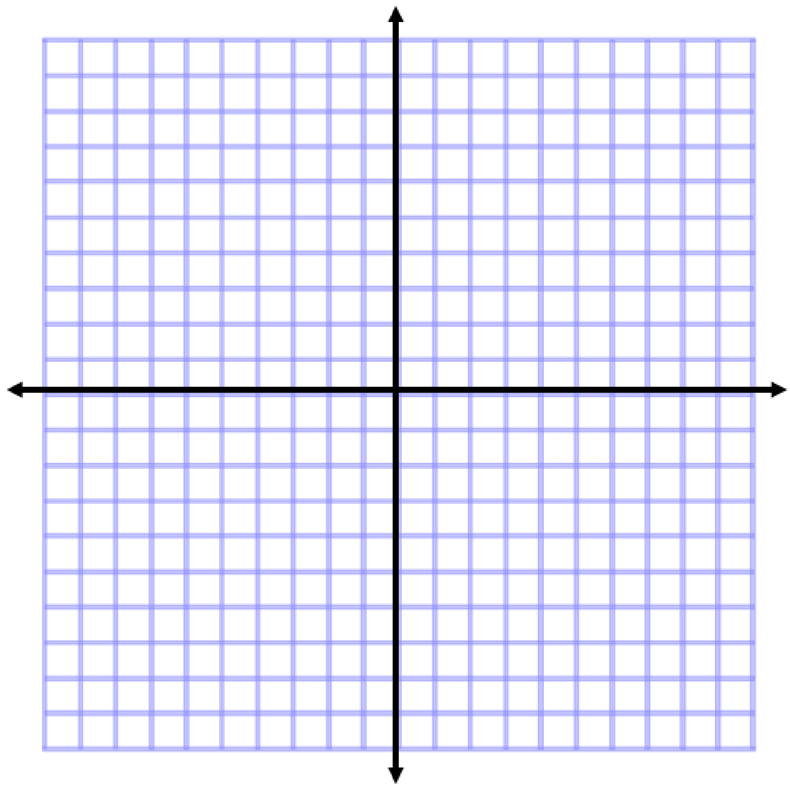
Name:______________

Graphing

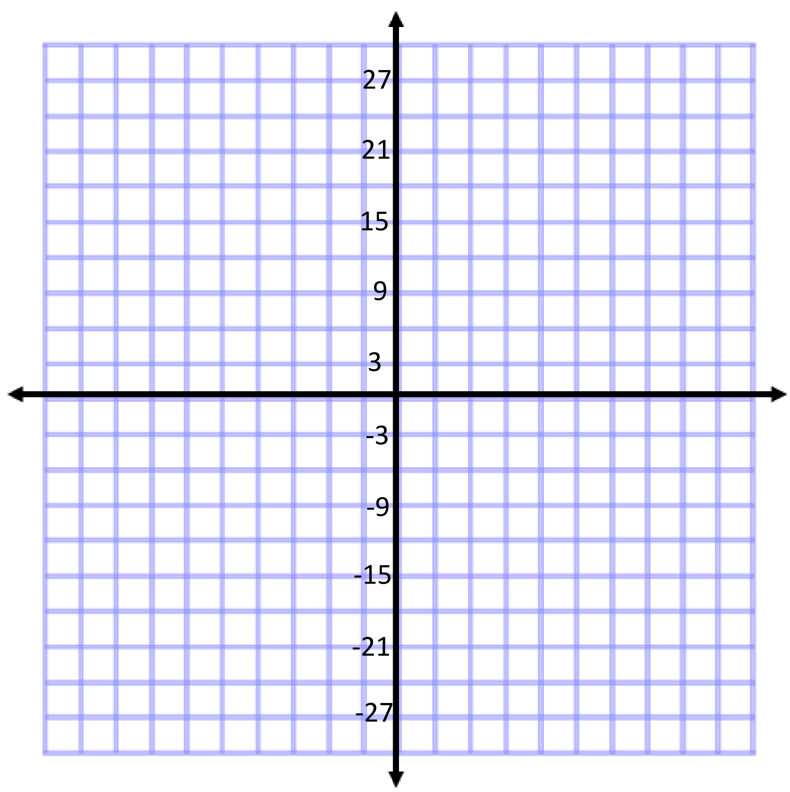
Learning Group:_______

Complete the square and graph the following quadratics with appropriate roots and vertex labeled. Also state the domain and range of the graphs

$$y = x^2 + 6x + 5$$



$$y = 2x^2 - 8x - 18$$



Week 2:

Class 1:

- SAT word of the day (5 min)
- Quick check-in (5 min)
  - Check-in over to find roots, graphing, domain/range
- Composition of functions (20 min)
  - Have them write down their favorite function that has a variable in it. Then have them plug in their favorite symbol
  - Write a couple of examples on the board
  - Now write your second favorite function and plug it into the first function. Any time you see your variable, plug the second function into the first function.
  - Now do it the other way around, plug the first function into the second function.
    - Are they the same? Hopefully not.
    - Why not?
  - These can be expressed as f(g(x)) or (f o g)(x)
  - Examples:
    - f(x)=3x, g(x)=2x+1, then switch them
    - f(x)=5x-2, g(x)=x^2+10, then switch them
    - You try f(x)=6x+ 4 , g(x)= 1/2 x+ 1/3
  - Then write the following functions on the board and have each of them shoot two functions. The ones they shoot are the ones they write down.
    - f(x)=x^2+3x
    - g(x)=3x+5
    - h(x)=7x-2
    - f(x)=4x-6
    - g(x)=x+12
    - h(x)=2x^2-5x
    - f(x)=5x+2
    - g(x)=12x-1
    - h(x)=6x^2+3
- Decomposition of functions (10 min)
  - If there's time, probably not.
- Math mission is "composite function worksheet"

Class 2:

- Quick check in on comp/decomp of functions (8 min)
- NEARPOD inverse functions (40 min)
  - What is an inverse function?
    - Graphically represent, how do we get this? What is it reflected about?
  - Take your x’s and your y’s and switch them. Solve for y.
  - Now let’s try it by taking points that are graphed and flipping the x’s and y’s. Then draw in the line.
  - Your final equation should be reflected about the y=x line.

Class 3: Need calculators and sticky graph paper

- Introduce transformations
  - First show y=x
  - Then show y=x+2. Because the +2 is there we shift it up 2.
  - Then show –x+2, because the negative is there, we change the slope, right?
  - Then 2x+2. How you're changing the slope.
- We represent these by function composition!
  - So if we had f(x)=x
    - We might say: f(x+1) to get f(x)=x+1
    - Or –f(x) to get f(x)=-x
    - Or f(x)-6 to get f(x)=x-6
    - Or 2f(x-5) to get f(x)=2x-10
  - There's a variety of different ones to use. HAND OUT CHEAT SHEET
    - GO OVER THE VARIETIES OF TRANSFORMATIONS using the following function
  - If we had $f(x)=x^2$ GRAPH EACH OF THESE IN YOUR NOTES
    - $f(x)+2= x^2+2$
    - $f(x)-4=x^2-4$
    - $f(x-3)=(x-3)^2$
    - $f(x+5)=(x+5)^2$
    - $2f(x)=2x^2$
    - $1/2f(x)=1/2x^2$
    - $-f(x)= -x^2$
    - $f(-x)=x^2$
- Do function dancing with x^2
- Make posters with partner
  - Use dart gun to shoot a target on the board. The group with the closest to the target gets to choose first
  - Use functions like: x^2, x^3, $x^{1/2}$, $x^{1/3}$, abs(x)
  - Make transformations that include -f(x), 2f(x), 1/2f(x), f(x)-4, f(x)+4, f(x-3), f(x+3)
- The sooner you get these done the sooner you can start your math missions.

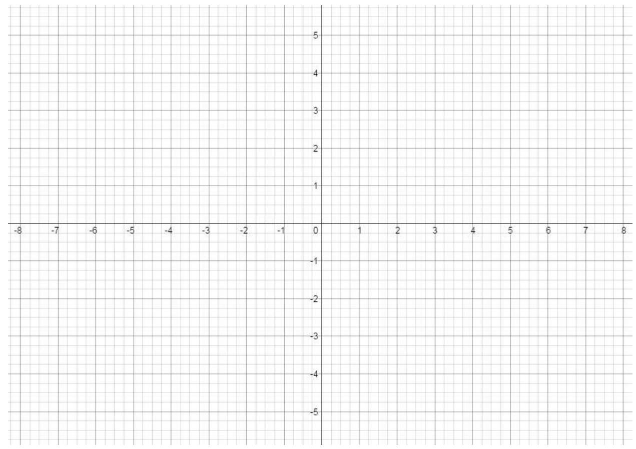
Cool activity:
http://images.pcmac.org/Uploads/JeffersonCountySchools/JeffersonCountySchools/Departments/DocumentsSubCategories/Documents/Math%20-%20Composition%20of%20Functions%20Relay%20-%20Chain%20Reaction%20Activity%20with%20Printable%20Cards.pdf

¡Glorious Inverses!

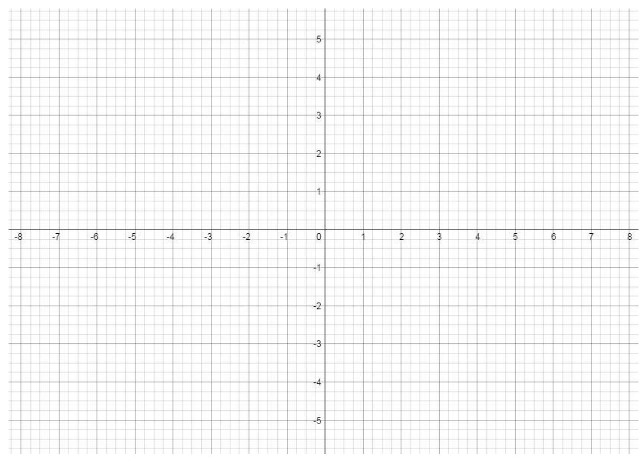
Name:_____________________ Learning Group:_______

For each of the following problems: (a) find the inverse of the function, (b) graph them both, (c) prove that they are, in fact, inverses.

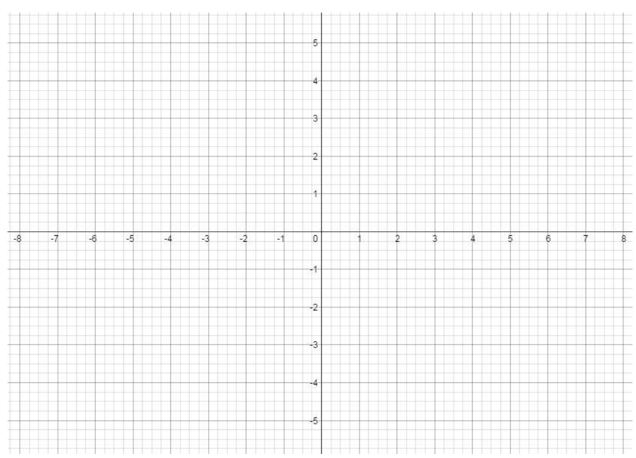
1. $f(x) = 2x - 1$



2. $h(x) = x^2 + 2$



3. $g(x) = 2(x+3)^3 - 1$



COMPOSITE FUNCTION WORKSHEET

Name:____________________________ Learning Group:_______

Directions: Show all work for credit. Work must be neat and answer must be circled.

For 1—6: Let $f(x) = 2x - 1$, $g(x) = 3x$, and $h(x) = x^2 + 1$.

1. $f(g(-3))$

2. $f(h(7))$

3. $(g \circ h)(24)$

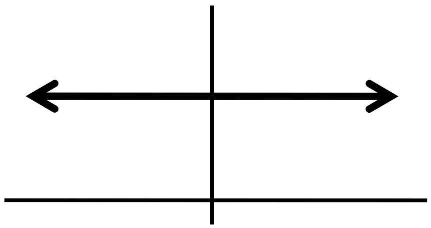
4. $f(g(h(2)))$

5. $h(g(f(5)))$

6. $g(f(h(-6)))$

7. You go into your favorite store equipped with a $10 off coupon and a 20% off coupon. You have an unlimited amount of money and purchase three items of your choice.
    a. List the three items you purchase and their total cost
    b. Create a function, f(x), to represent the $10 off coupon
    c. Create a function, g(x), to represent the 20% off coupon
    d. Evaluate f(g(x)) and g(f(x)). Which one is better in terms of your final cost?

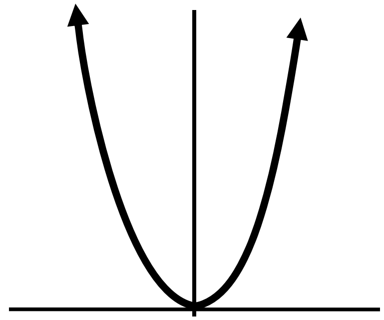
$f(x) = \text{a number}$



Domain: $(-\infty, \infty)$

Range: the number

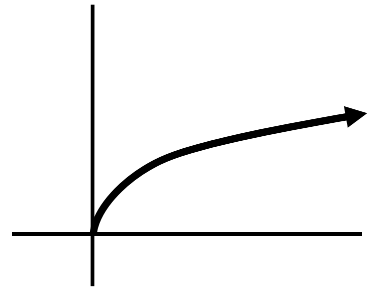
Parabola: $f(x) = x^2$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

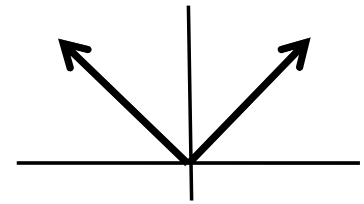
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$

Range: $[0, \infty)$

$f(x) = |x|$

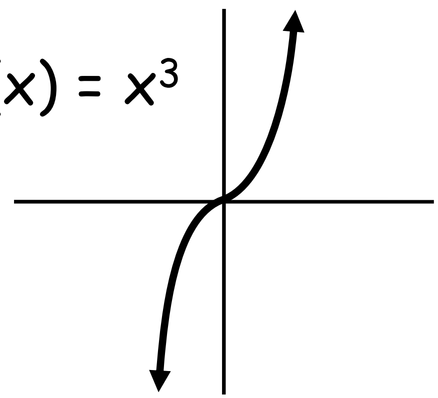


Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Library Function Graphs

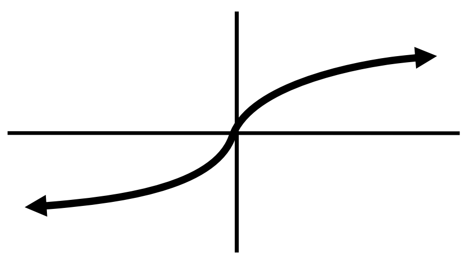
$f(x) = x^3$



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

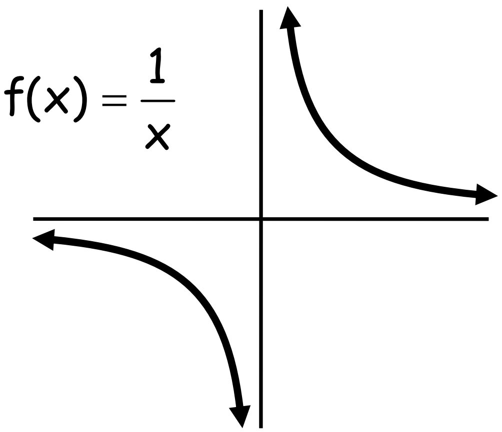
$f(x) = \sqrt[3]{x}$



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

$f(x) = \frac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Graphs and Transformations

Name: ____________________________ Learning Group:_______

Evaluate each transformation in the function, then graph each—be sure to label all of them!

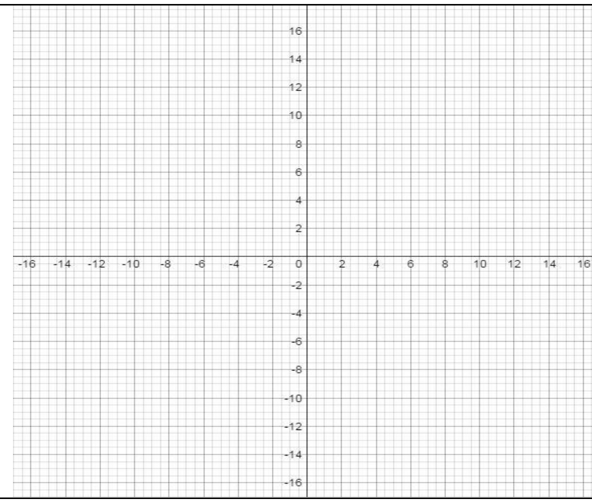
$f(x) = x + 1$

$f(x-4) =$

$f(x+8) =$

$f(-x) =$

$-f(x) =$



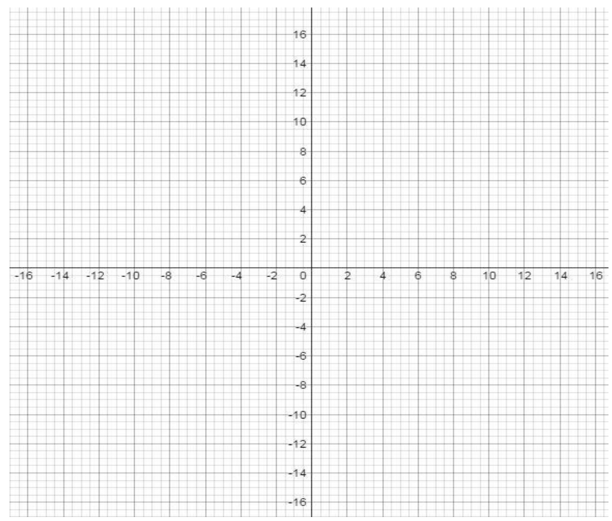
$f(x) = x^2$

$\frac{1}{4} f(x) =$

$f(x-10) =$

$-f(x) - 6 =$

$-\frac{1}{8} f(x) =$



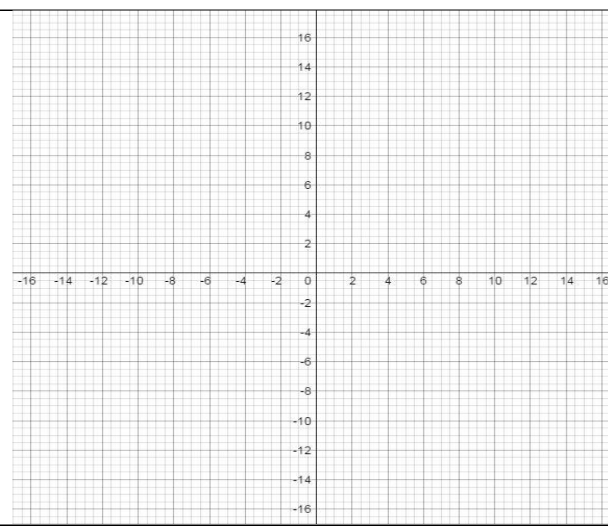
$f(x) = \sqrt{x}$

$f(x) - 10 =$

$f(-x) + 5 =$

$f(x-6) =$

$4f(x+7) =$

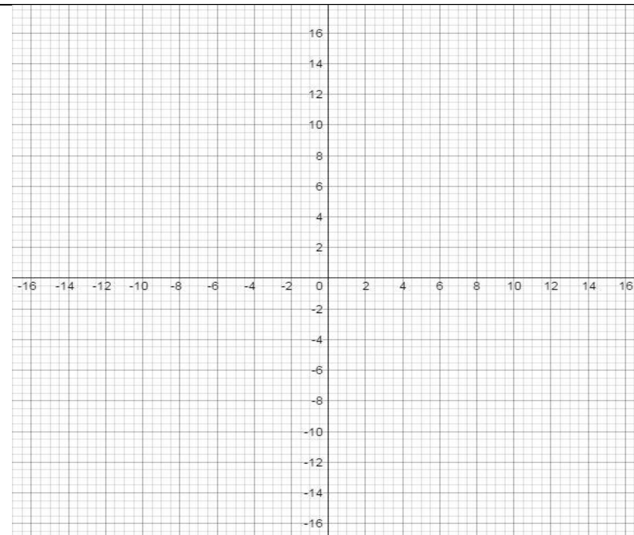


$f(x) = x^3$

$\frac{1}{8} f(x) =$

$-f(x+7) =$

$f(x-8)-9 =$

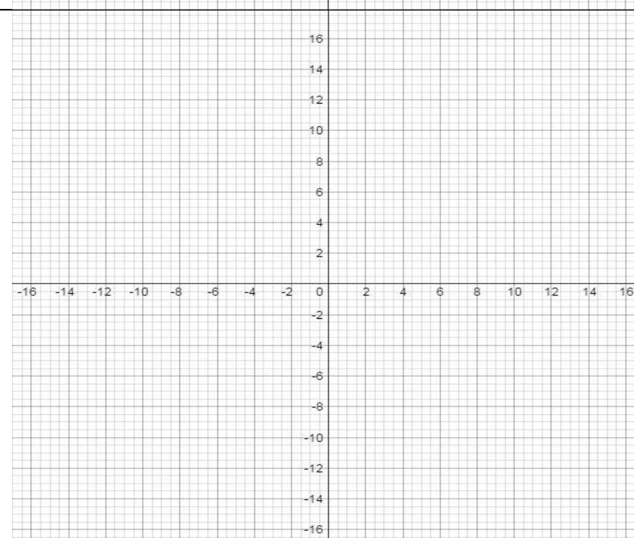


$f(x) = |x|$

$2f(x) =$

$-f(x+8)+3 =$

$f(x-6)-4 =$

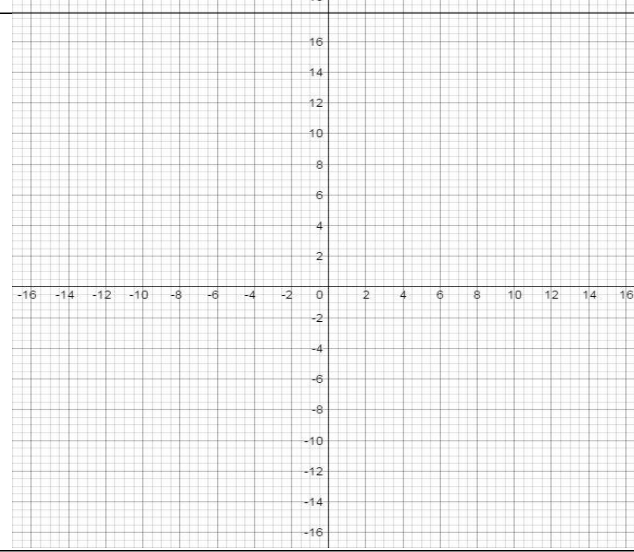


$f(x) = \sqrt[3]{x}$

$f(-x)+10 =$

$4f(x+5) =$

$f(x-7)-9 =$



Graphs and Transformations

Name: ____________________________ Learning Group:_______

Evaluate each transformation in the function, then graph each—be sure to label all of them!

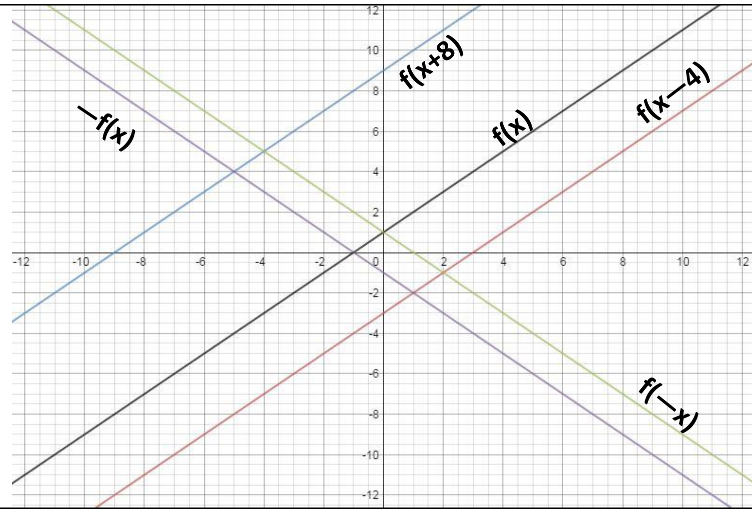
$$f(x) = x + 1$$

$$f(x-4) = x - 3$$

$$f(x+8) = x + 9$$

$$f(-x) = -x + 1$$

$$-f(x) = -x - 1$$



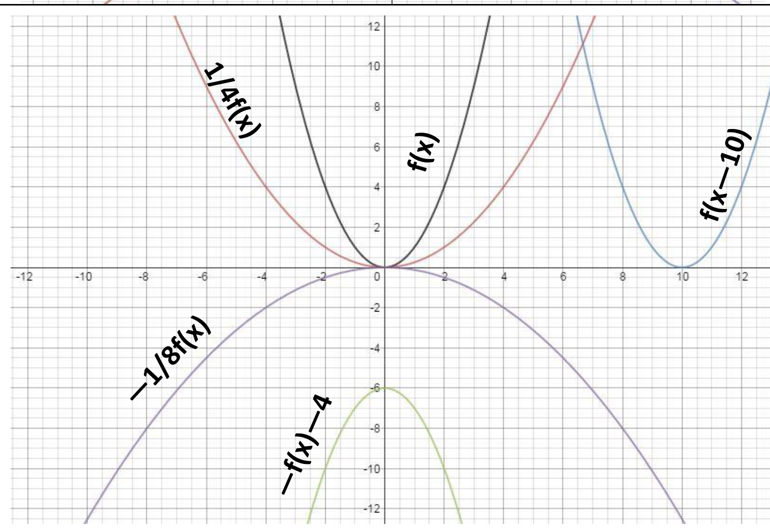
$$f(x) = x^2$$

$$\frac{1}{4}f(x) = \frac{1}{4}x^2$$

$$f(x-10) = (x-10)^2$$

$$-f(x) - 6 = -x^2 - 6$$

$$-\frac{1}{8}f(x) = -\frac{1}{8}x^2$$



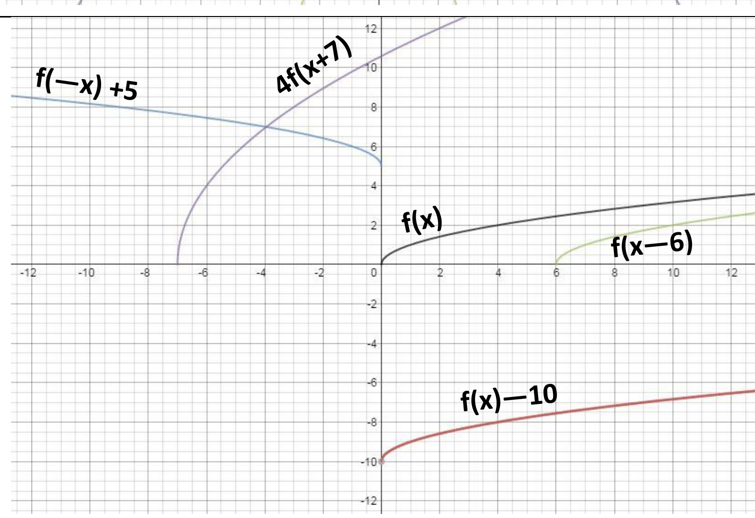
$$f(x) = \sqrt{x}$$

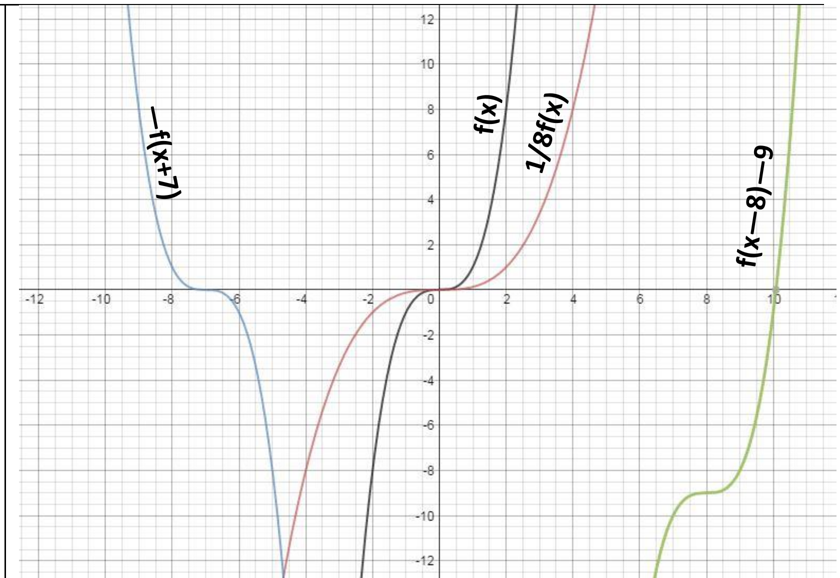
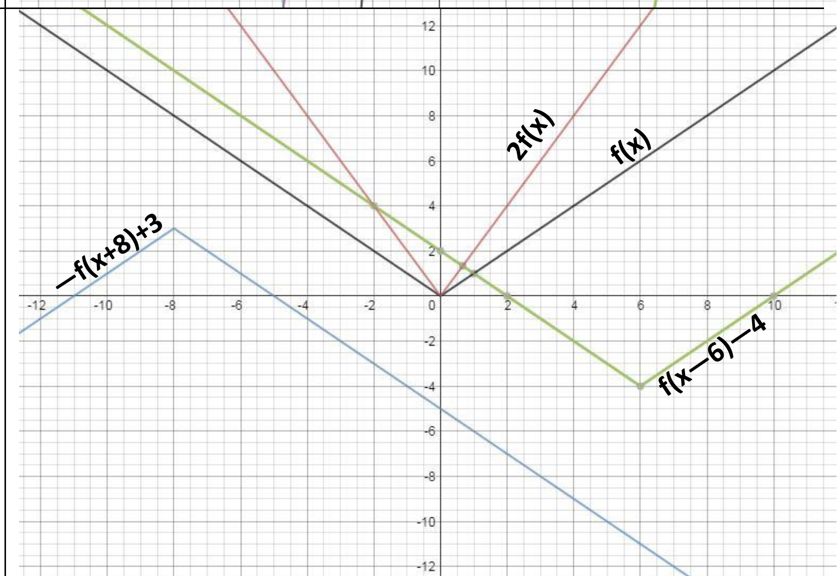
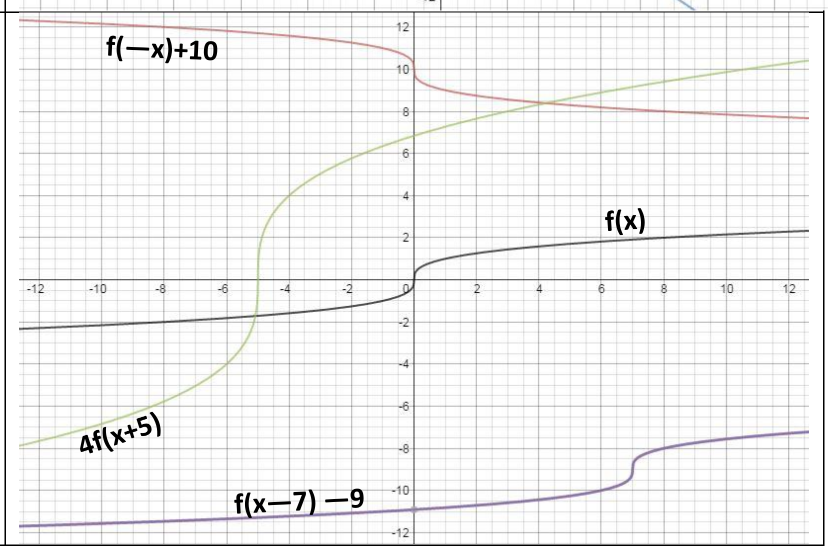
$$f(x) - 10 = \sqrt{x} - 10$$

$$f(-x) + 5 = \sqrt{-x} + 5$$

$$f(x-6) = \sqrt{x-6}$$

$$4f(x+7) = 4\sqrt{x+7}$$



$f(x)=x^3$ $\frac{1}{8}f(x)=\frac{1}{8}x^3$ $-f(x+7)=-(x+7)^3$ $f(x-8)-9=(x-8)^3-9$	
$f(x)=\lvert x\rvert$ $2f(x)=2\lvert x\rvert$ $-f(x+8)+3=-\lvert x+8\rvert+3$ $f(x-6)-4=\lvert x-6\rvert-4$	
$f(x)=\sqrt[3]{x}$ $f(-x)+10=\sqrt[3]{-x}+10$ $4f(x+5)=4\sqrt[3]{x+5}$ $f(x-7)-9=\sqrt[3]{x-7}-9$	

QUICK CHECK IN

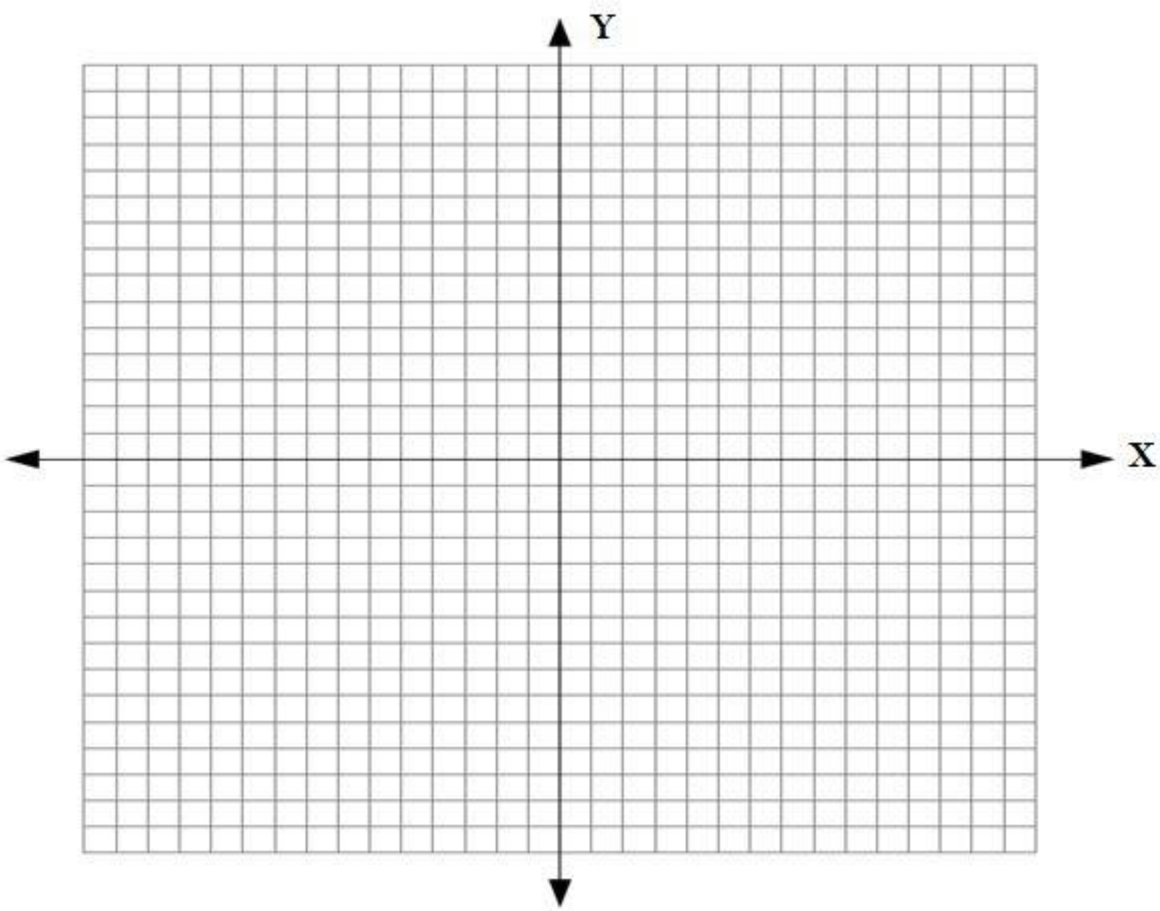
Name:______________________ Learning Group:_______

Here is a check-in to test your knowledge, don't get behind or else you'll fall…edge?

1. Find the inverse of the following function.

2. Graph them both

$$y = (x-2)^2 + 5$$



Week 3:

Class 1: Vertical Asymptotes/Removable Discontinuities

- SAT QOD (5 min)
- Homework Questions (5 min)
- Check in (8 min)
- What is a hole or asymptote? (30 min)
- Hand out calculators. Have them graph along with you.
    - What produces a vertical asymptote?
        - Zeros in the denominator
        - Thus far we've only dealt with functions that are pretty much f(x) or g(x) or f(x)g(x), but what about f(x)/g(x). We'll take one polynomial and divide it by another polynomial.
        - So when we do that, there will almost always be an instance where there is a zero in the denominator. What happens when you divide by zero? Well, really bad things.
        - That is where the function is undefined. That is where we see vertical asymptotes.
        - Let's try 1/x, what values will the function be undefined? 0! Let's plug in values to the left and the right. MAKE A TABLE
        - Let's try $1/x^2$, what values will make the function undefined? 0! Make a table! COOL HUH?
        - What if there is more than one asymptote? How about $1/(x^2+3x+2)$? Factor, find there are asymptotes at x=-1, and -2.
        - Ok, we got the idea. How about this? $(x^2+2x-3)/(x^2-5x-6)$. Well, they are both factorable, so let's do it. Numerator is (x+3)(x-1) and the denominator is (x-6)(x+1). Nothing can cancel, so we know there are two asymptotes at 6, and -1.
        - Well what if we can get something to cancel? Then we get a removable discontinuity. Let's try $(x+1)/(x^2-5x-6)$ to get (x+1)/(x+1)(x-6).
        - Or how about $(x^2+3x+2)/(x+2)$ to get (x+2)(x+1)/(x+2).
        - This makes a hole, it doesn't go away… it's still there!

Class 2: Horizontal/Slant Asymptotes

- SAT QOD (5 min)
- Mission Questions (5 min)
  - What produces a horizontal asymptote? Write this on the board
    - **When the degree in the numerator is the same as the denominator**, we take the leading terms and divide them.
    - **When the degree of the denominator is greater than the degree of the numerator**, as know that the denominator is 'pulling' the function. What happens when the denominator gets really big? The fraction gets really small. So we have a horizontal asymptote at y=0.
  - What produces slant asymptotes?
    - Well, for your mission you only did problems where the denominator divided evenly into the numerator. Now how about ones where they don't.
    - **When the degree of the numerator is larger than the degree of the denominator, do long division.** If it goes in evenly, we have a removable discontinuity. If it does not, we have the quotient as the slant asymptote
    - This is also known as your "end behavior"
  - Try these: Use your white boards, but then… BE SURE TO COPY INTO NOTES!
    - $(x^2-6x+7)/(x+5)$
    - $(x^2-x-6)/(x^2-1)$
    - $(3x^2+x-2)/(2x+6)$
    - $(x+3x^2)/(5x^2-6x-1)$
    - $(5x^3-8)/(x^2+3x-1)$
    - $(2x)/(x^2-5x-3)$
  - Mission, continue on the paper!
  - Show video if there's time

Class 3: Graphing on Paper/Introduce Unit Circle

- SAT QOD (5 min)
- Mission Questions (5 min)
- Graph functions on paper, find domain, range, label any removable discontinuities, asymptotes (vertical and horizontal), and state the end behavior. Do graphs each.
    - $(x-3)/(x^2+x-12)$
    - $(3x-1)/(x-5)$
    - $(3x^2+6x+5)/(x^2-3x+2)$
    - $(x+6)/(x^2-5x)$
    - $(x^2-2x)/(x+4)$
    - $(x^2-9x+18)/(x^2-5x-6)$
    - $(x-7)/4x$
    - $(x+12)/(5x^2-10x)$
    - $(3x^2-9)/(x^2+7x+12)$
- Introduce Unit Circle, pi! (If there's time)
    - Give them a blank one. Their mission is to finish the graphing paper and fill in the unit circle. They can look it up online, they don't need to do it all out.
    - Basically explain what a radian is and how to find it. Using pi/180
        - First let's fill in the degrees of the unit circle
        - Second we'll find the radians for the first and third quadrants, they need to do the rest
        - Start mission with remaining time or show video!

Asymptotes of Rational Functions

Name:________________________________

Identify all vertical asymptotes for each function.

1. $f(x)=\dfrac{5x}{x-1}$

2. $f(x)=\dfrac{3x^2}{x^2-1}$

3. $f(x)=\dfrac{3x^2+x-5}{x^2+1}$

4. $f(x)=1-\dfrac{3}{x-3}$

5. $f(x)=\dfrac{x^2-5x+4}{x^2-4}$

6. $f(x)=\dfrac{x^3}{2x^2-8}$

Determine whether the graph will have a horizontal or a slant asymptote, then find it.

7. $f(x)=\dfrac{3x^2+1}{x^2+x+9}$

8. $f(x)=\dfrac{4}{(x-2)^3}$

9. $f(x)=2+\dfrac{5}{x^2+2}$

10. $f(x)=\dfrac{x^2+1}{x}$

11. $f(x)=\dfrac{2x^2-5x+5}{x-2}$

12. $f(x)=\dfrac{2x^3-x^2-2x+1}{x^2+3x+2}$

Based on the asymptotes, match each equation with its graph. (Don't use a calculator!)

____13. $f(x)=\dfrac{1}{x-1}$

____14. $f(x)=\dfrac{5x}{x-1}$

____15. $f(x)=\dfrac{3x^2}{x^2-1}$

____16. $f(x)=\dfrac{4x}{x^2-1}$

____17. $f(x)=-\dfrac{x+2}{x+4}$

____18. $f(x)=\dfrac{x-1}{x-4}$

____19. $f(x)=\dfrac{x+1}{x-3}$

____20. $f(x)=\dfrac{2x}{x-3}$

____21. $f(x)=\dfrac{1-x^2}{x}$

____22. $f(x)=\dfrac{x^2-3x+2}{x}$

____23. $f(x)=\dfrac{1+3x^2-x^3}{x^2}$

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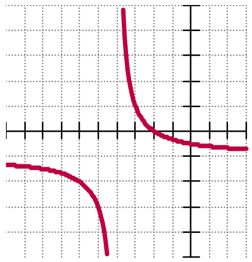

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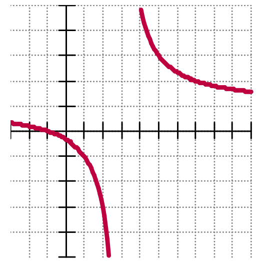

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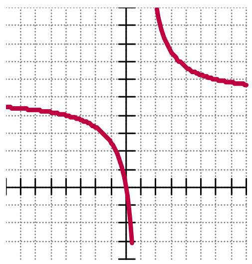

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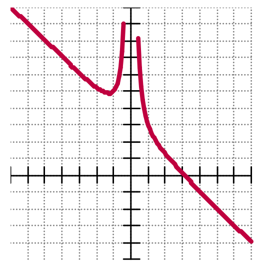

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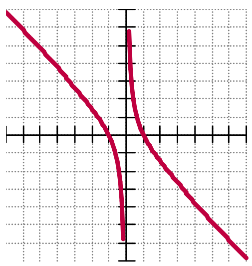

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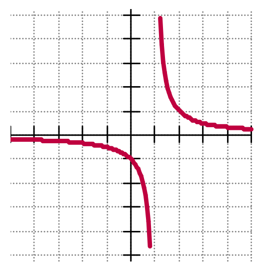

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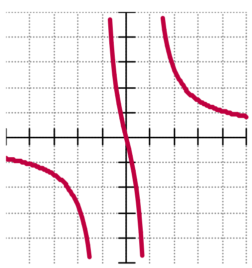

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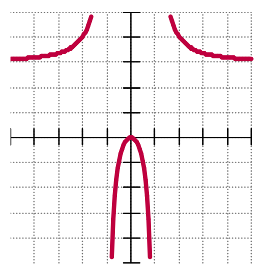

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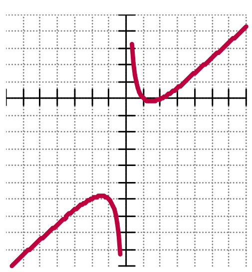

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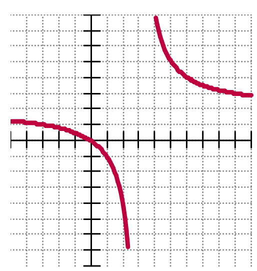

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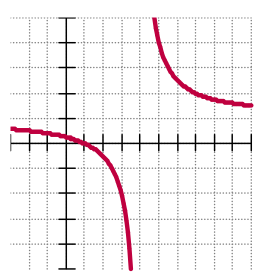
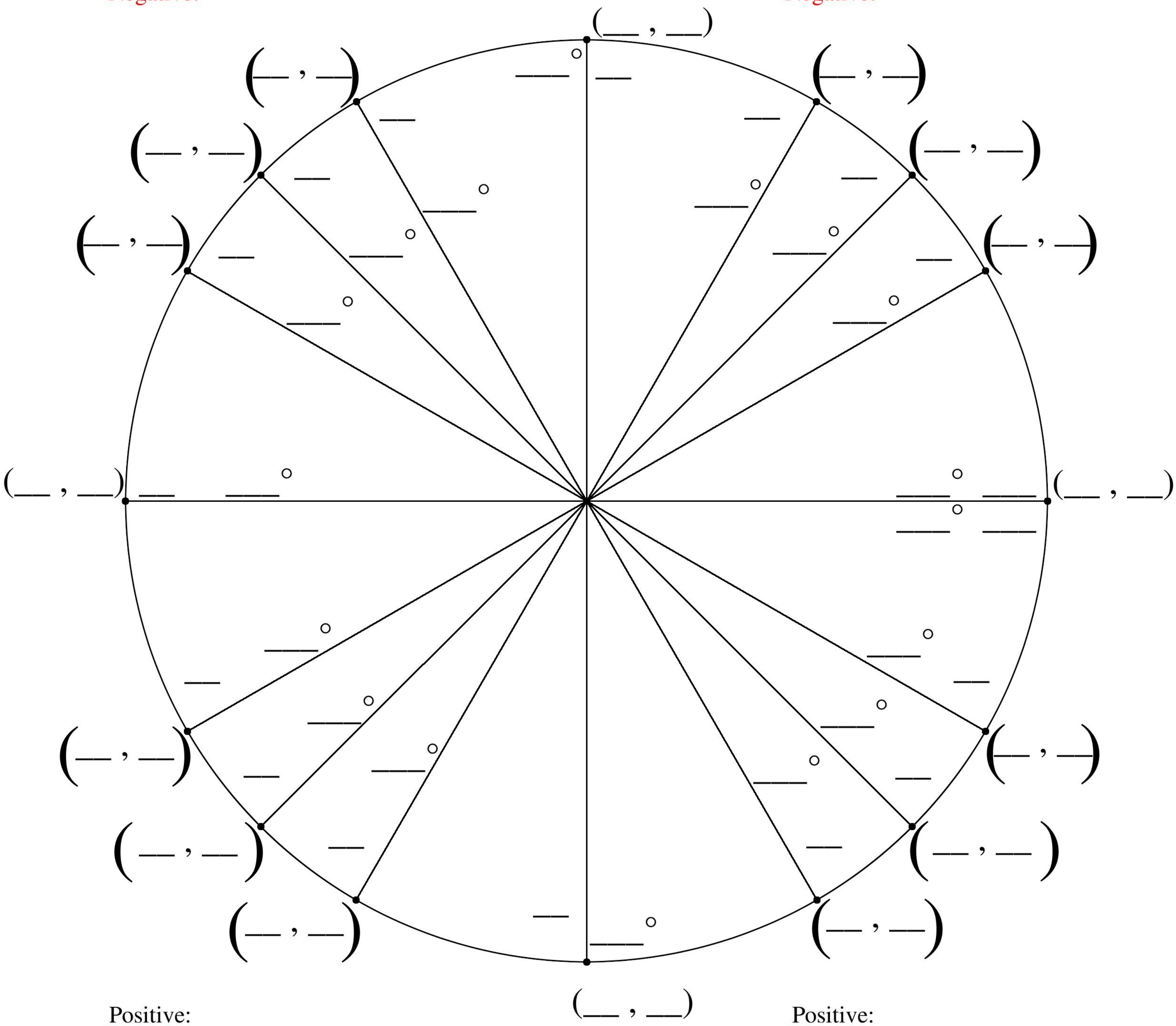


Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:



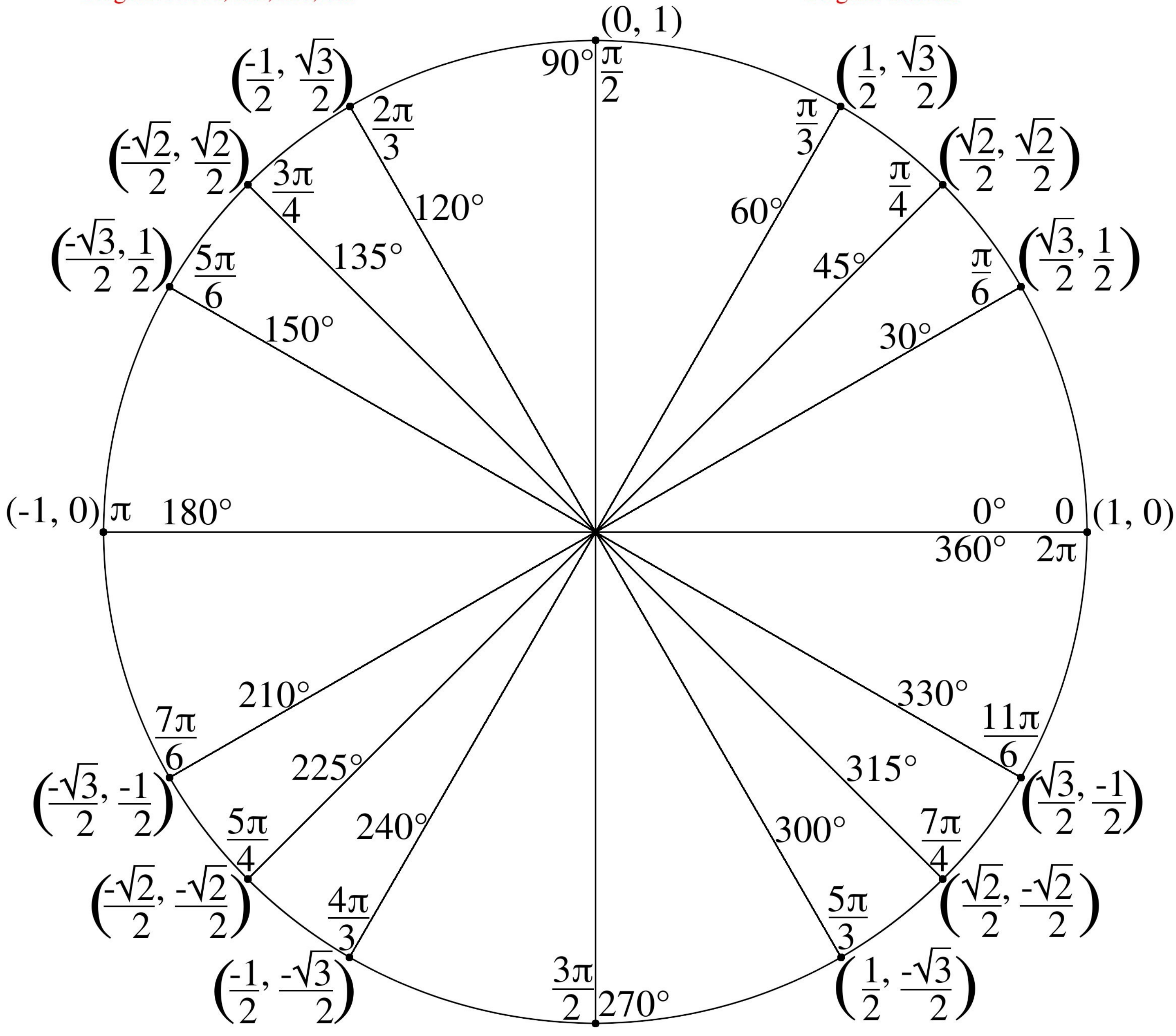
Positive:
Negative:

Positive:
Negative:

The Unit Circle

Positive: sin, csc
Negative: cos, tan, sec, cot

Positive: sin, cos, tan, sec, csc, cot
Negative: none



Positive: tan, cot
Negative: sin, cos, sec, csc

Positive: cos, sec
Negative: sin, tan, csc, cot

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