Week 4:

Day 1: Unit circle and meaning!

- SAT QOD (5 min)
- Mission Questions (10 min)
  - Should lead right into unit circle and what the heck it means
- It begins! (20 min)
  - The unit circle has radius 1.
  - Fill it in based on degrees first
    - Start with 360, half is 180, half is 90, 90 plus 180 is 270. Boom.
    - Then go for the smaller guns, half of 90 is 45, 45 plus 90 is 135, 45 plus 180 is 270 plus 45 is 315, 315 plus 45 is 360. Boom.
    - Then do the other ones, starting with 30 plus 30 is 60 plus 30 is 90, go from there. Boom.
  - Let's do conversion from degrees to radians.
    - What is a radian? A radian is a unit of measure for the arc of the circle. Based on the circumference. What is the circumference of a circle? 2pir. You've used radians already and you didn't even know it. This is because you're taking the number of radians it takes to go around the circle and multiplying it by the radius to give it the circumference of a specific circle.
    - So, to convert we use the formula **radians=pi/180.** So we'll do the first two quadrants, take about 10 minutes to do the rest. Be sure that they are in simplest form.
- Now we'll find the points around the unit circle (20 min)
  - Start with the 60 degree one. Devise the triangle and use Pythagoras to find the side lengths. This is just like a Cartesian coordinate plane.
  - Show desmos with unit circle and how we can plot all the points. BE SURE TO SCROLL DOWN FROM THE EQUATION OF THE CIRCLE SO IT DOESN'T CONFUSE THEM!
  - Now show the 45 degree one. A little trickier.
- Ok, now with a partner to complete the unit circle puzzle.
  - Take graph paper and have them draw a circle and place the puzzle pieces where they go.
- Talk about mission…Watching video and bringing notes:
  - https://www.khanacademy.org/math/trigonometry/trig-function-graphs/trig_graphs_tutorial/v/we-graph-domain-and-range-of-sine-function

Day 2:

- SAT QOD (5 min)
- Mission Questions (5 min)
- Create the graphs of sin(x), cos(x), and tan(x) by evaluating at each point on the circle (30 min)
    - Groups of two will get together and graph cos(x) and tan(x) together
    - They need to use the radians as their x and the sin(x) or cos(x) as their y values
    - They can convert them to actual numbers if they'd like
    - Have them give you their points and plot them first
    - Then graph the sin and cos graph to show they go through the points they found
- Give them blank unit circle to fill in as best they can without their notes, then have them use their notes (10 min)
- Start mission

Day 3: More trig stuff

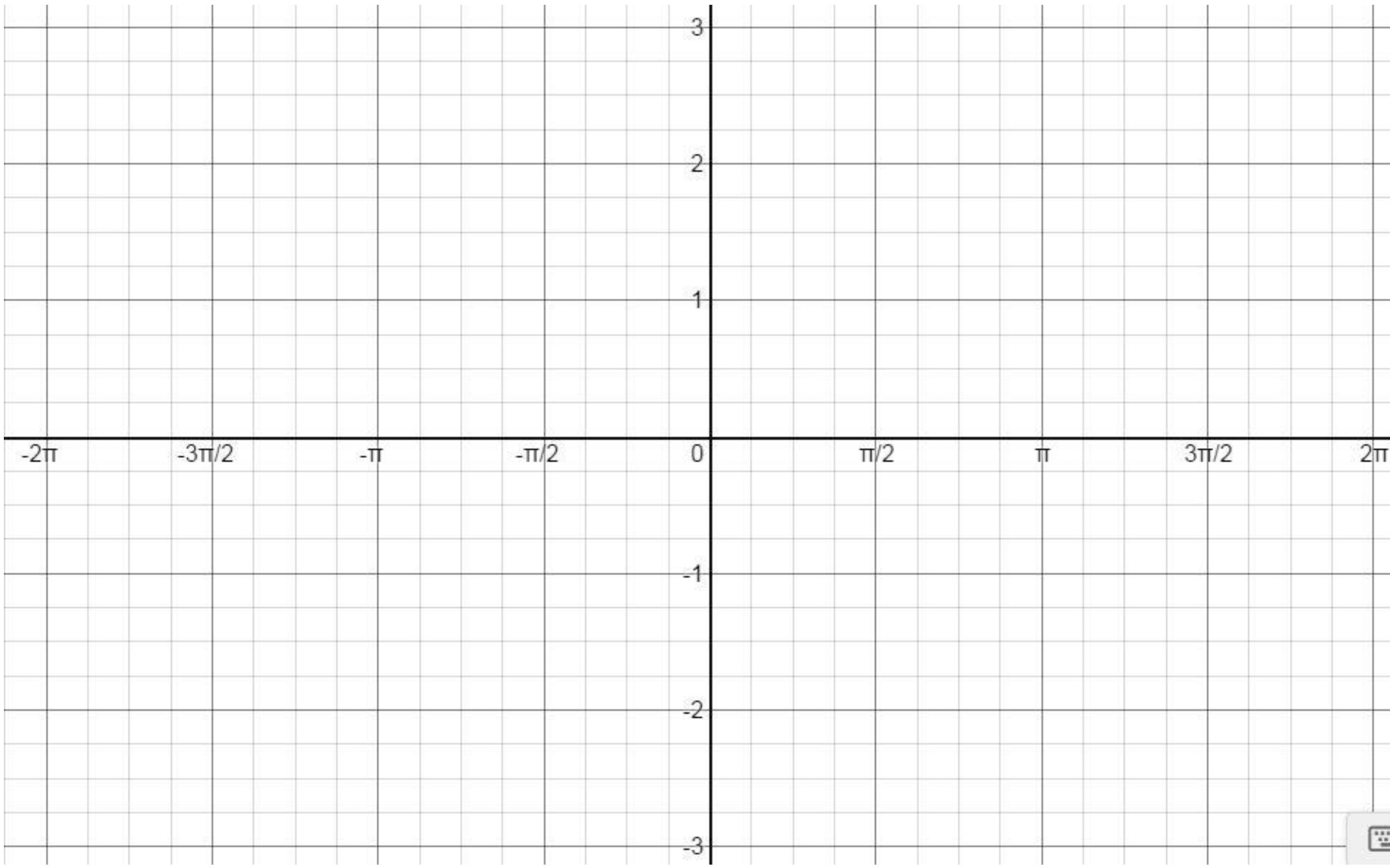
- SAT QOD (5 min)
- Mission Questions (5 min)
- Finish tangent graph (10 min)
- Show them trick with filling in the unit circle (20 min)
    - Counting radians around
    - Show them hand trick for unit circle
    - Fill in a blank one
        - USE A PENCIL
- Have them work in pairs to find the answers to the following trig questions (rest of class)
    - If they get it correct, they have a chance to shoot the board and hit the target. If they hit it they get plus 5 points.
    - Need team names!
        - 60 degrees = how many radians?
            - Pi/3
        - 225 degrees = how many radians?
            - 5pi/4
        - 150 degrees = how many radians?
            - 5pi/6
        - sin(pi/4)
            - root(2)/2
        - cos(2pi/3)
            - -1/2
        - tan(pi/3)
            - root(3)
        - cos(5pi/6)
            - -root(3)/2
        - sin(5pi/4)
            - -root(2)/2
        - tan(3pi/2)
            - und
        - tan(pi)
            - 0
        - sin(3pi/2)
            - -1
        - cos(7pi/4)
            - root(2)/2
        - sin(5pi/3)
            - -root(3)/2
        - tan(11pi/6)
            - -1/root(3)
- Mission: Bring in any questions you have about the unit circle

UNIT CIRCLE STUFF YAAS

Name:______________________ Learning Group:_______

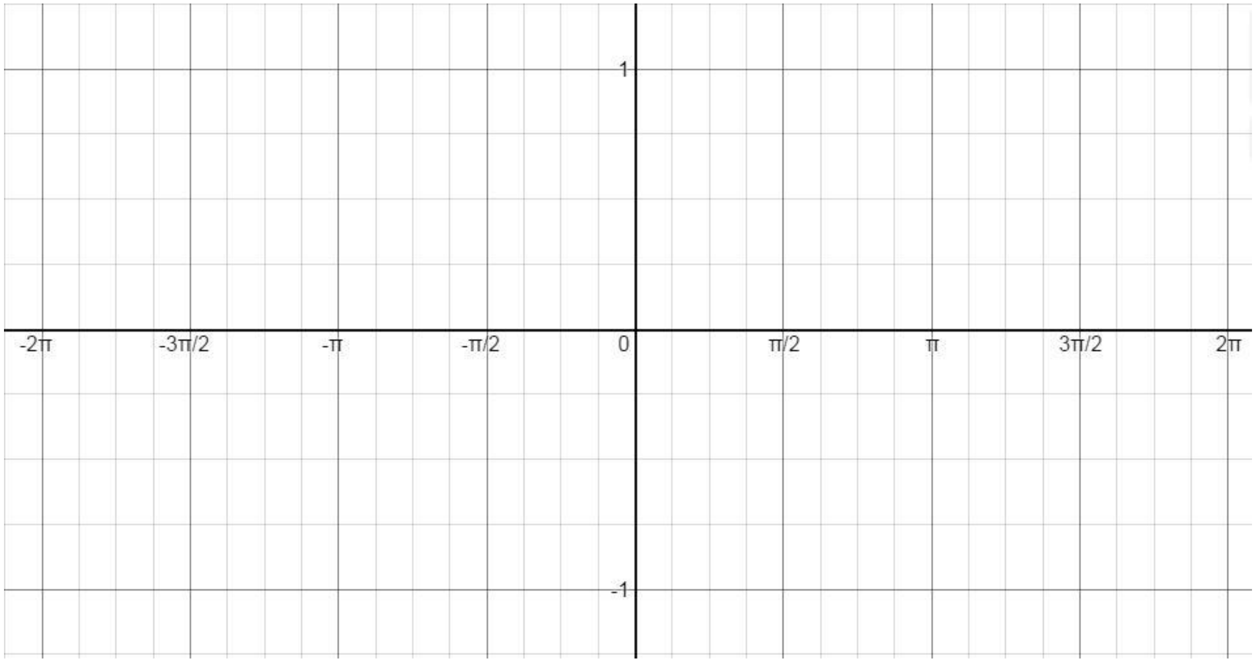
Evaluate each of the following:

1. $\sin(\frac{\pi}{3})$	2. $\cos(\frac{\pi}{6})$
3. $\tan(\frac{\pi}{4})$	4. $\sin(\frac{3\pi}{4})$
5. $\cos(\frac{5\pi}{6})$	6. $\tan(\frac{2\pi}{3})$
7. $\sin(\frac{7\pi}{6})$	8. $\cos(\frac{11\pi}{6})$
9. $\tan(\frac{7\pi}{4})$	10. $\sec(\frac{\pi}{4})$



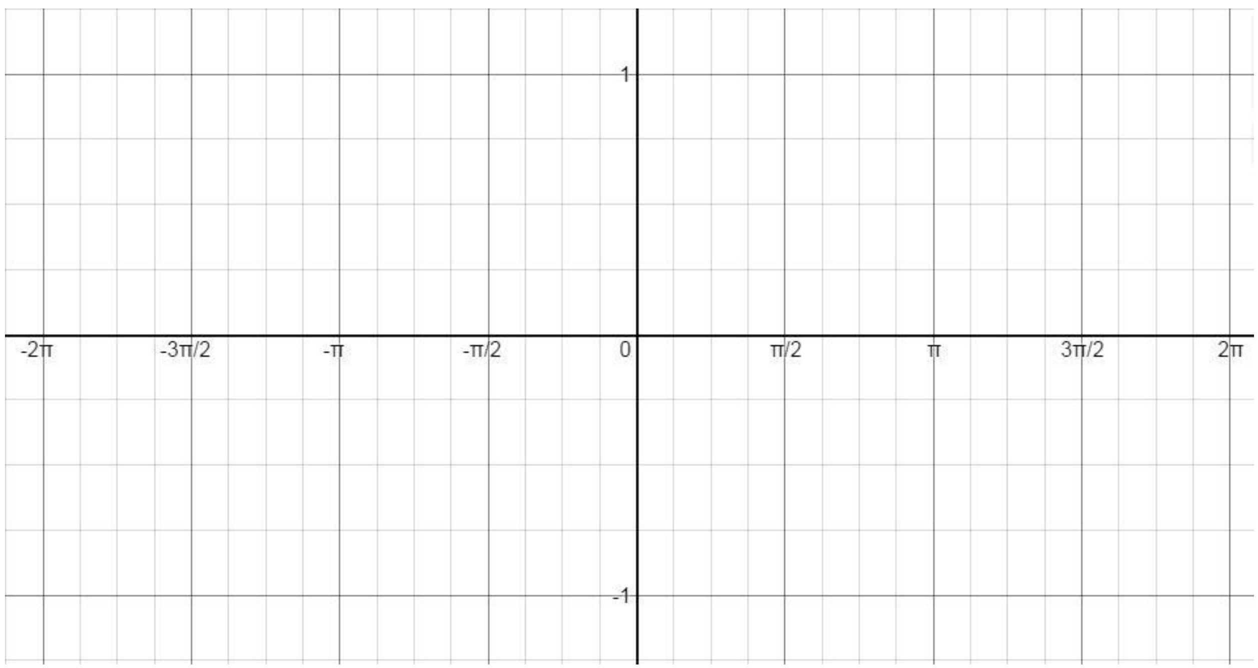
X	tan(x)

$y=sin(x)$



x	sin(x)

$y=cos(x)$



x	cos(x)

$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
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$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
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$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
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$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
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$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$(0,1)$	$(0,1)$	$(0,1)$	$(0,1)$	$(0,1)$
$(-1,0)$	$(-1,0)$	$(-1,0)$	$(-1,0)$	$(-1,0)$
$(0,-1)$	$(0,-1)$	$(0,-1)$	$(0,-1)$	$(0,-1)$

$\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{6}$
$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$
$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$
$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$
$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$
$\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$
π	π	π	π	π
$\frac{7\pi}{6}$	$\frac{7\pi}{6}$	$\frac{7\pi}{6}$	$\frac{7\pi}{6}$	$\frac{7\pi}{6}$
$\frac{5\pi}{4}$	$\frac{5\pi}{4}$	$\frac{5\pi}{4}$	$\frac{5\pi}{4}$	$\frac{5\pi}{4}$
$\frac{4\pi}{3}$	$\frac{4\pi}{3}$	$\frac{4\pi}{3}$	$\frac{4\pi}{3}$	$\frac{4\pi}{3}$
$\frac{3\pi}{2}$	$\frac{3\pi}{2}$	$\frac{3\pi}{2}$	$\frac{3\pi}{2}$	$\frac{3\pi}{2}$
$\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$
$\frac{7\pi}{4}$	$\frac{7\pi}{4}$	$\frac{7\pi}{4}$	$\frac{7\pi}{4}$	$\frac{7\pi}{4}$
$\frac{11\pi}{6}$	$\frac{11\pi}{6}$	$\frac{11\pi}{6}$	$\frac{11\pi}{6}$	$\frac{11\pi}{6}$
2π	2π	2π	2π	2π

Week 5:

Class 1: Graphing sin(x) and cos(x) like bosses
Great resource: http://www.mathsisfun.com/algebra/amplitude-period-frequency-phase-shift.html
Go to the CC if possible: use desmos
Seat accordingly: (Tresten, Sam), (Alec, Jasmine), (Megan, Ashlynn), (Brandon, Nicole), (Travis, Faith-Anne) and (Mitch, Cassy), (Gavin, Keristin)

- DO NOT SIGN ONTO COMPUTERS YET
- Mission Questions (5 min)
- First plot what sin(x) and cos(x) look like. Be sure to stress that we only need critical points. Once we plot those we can draw it in.
- Talk about period, amplitude, phase shift, etc… (Class)
    - Examples and problems with what each one does
        - Have them use their white boards to sketch so it is easily erased, then copy down in notes when it is correct.
        - Start with graphing what sin(x) and cos(x) look like. Where the very key points are: 0, pi/2, pi, 3pi/2, 2pi
    - Vertical shift: We know the range is from -1,1.
        - So what if we have sin(x)+1? This shifts the graph up one on the y axis, similar to what we did earlier with transformations.
        - What about cos(x)-4
        - Now you graph
            - sin(x)+6
            - cos(x)-1
    - What about amplitude?
        - Remember the graphs range from -1,1
        - It will be like how tall a wave is off the water.
        - 2sin(x) will multiply your function's height by 2
        - 3cos(x) will multiply it by 3
        - Now you graph:
            - 4cos(x)
            - 1/2sin(x)
    - Now graph 2sin(x)—5
    - What about the phase shift?
        - This is the same idea as with the other transformations we did
        - It's inside the parenthesis so it moves the graph left to right and vice versa
        - But, we need to say that it is –c/b, because it is dependent on the period
        - So sin(x+pi/2) would be shifted to the left pi/2, and that's there I put a point to start from. I know my other points, in this situation, are +pi/2 from there, +pi, +3pi/2, and plus 2pi.
            - sin(x+pi) = phase shift of -pi, then -pi/2, 0, +pi/2, +pi

- How about the period?
  - This means how many cycles can I fit into the normal 2pi cycle
  - For sin(x) and cos(x) how many fit in? 1!
  - What about sin(2x)? 2!
  - So how do you plot those points? Take 2pi/2, and you get pi. So that should be the end of one cycle. To find the points, we need to know the **frequency**, which is the reciprocal of the period. So it is ½ your normal points. So normally we plot pi/2, pi, 3pi/2, and 2pi. Instead we will plot pi/2 divided by 2, or pi/4, pi/2, 3pi/4, and pi.
- Give them the worksheet and have them sign on.

- **Steps for graphing:**
  - **First identify the vertical shift**
  - **Then identify the amplitude**
  - **Then find new points by multiplying normal points by the frequency**
  - **Add those to the phase shift**
  - **Identify new y values**
  - **Plot**

Class 2:

- Mission Questions (5 min)
- Check in/review for what they learned the night before (10 min)
- Project for graphing sin or cos graph (all class)
    - In the computer center have them graph the functions they get on their computers and then let them use their artistic skills to create the sinocoaster on the graph paper using the points on the graph on the computer
    - DON'T FORGET ABOUT THE PHASE SHIFT
- The sinocoaster
- If they finish
    - Make your own unit circle with your favorite circular picture in desmos
    - Print it out to keep!
- Mission, go through all of your pre calc stuff, organize, and bring more questions about the summer to class.

Class 3: Need folders! REVIEW DAY

- Mission Questions (5 min)
- Start review/making things that will help them remember everything they learned
    - Go through function composition
    - Go through inverses
    - Go through rational function bible
- START JEOPARDY
    - Get in two teams of three and a team of four
- http://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Parent_Functions_Cheat_Sheet_2014.pdf

Pre-Calc—Periodic Functions Transformations

Name:______________________________ Learning Group:_______

1. Go to Desmos Graphing Calculator
2. Type in one of the functions below and add sliders to each letter. Set the sliders from $-5 \le x \le 5$
3. For whichever function you choose, enter sin(x) or cos(x) to correspond with it with a dotted line
4. Experiment with the sliders to see what happens with each of the variables

asin(bx+c)+d

acos(bx+c)+d

a = amplitude

$\frac{2\pi}{b}$ = period

$-\frac{c}{b}$ = phase shift

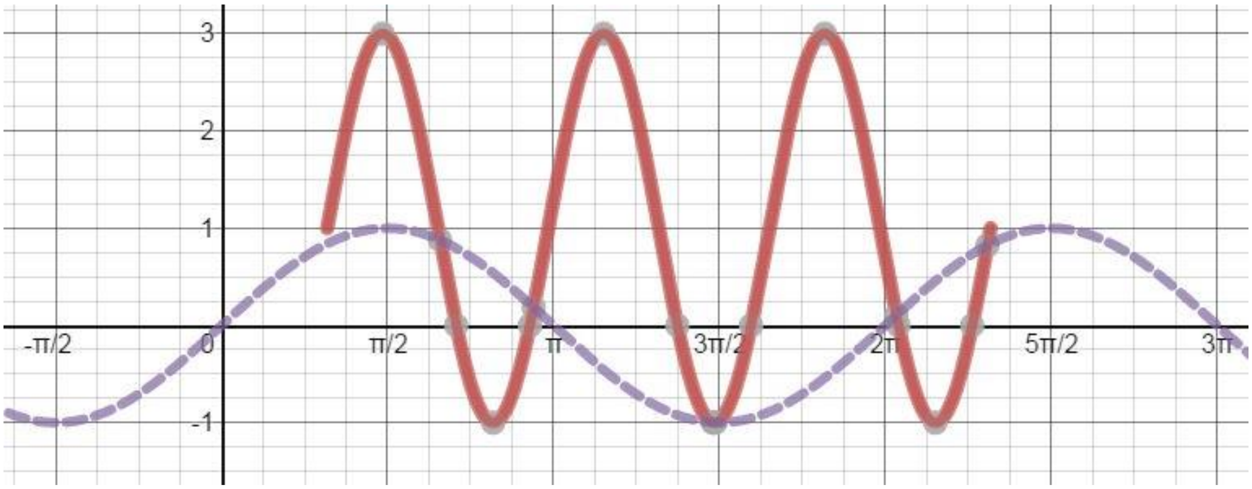
d = vertical shift

As the **amplitude** *increases*, the graph____________________________, and as the **amplitude** *decreases*, the graph____________________________

As the **period** *increases*, the graph__________________________, and as the **period** *decreases*, the graph__________________________

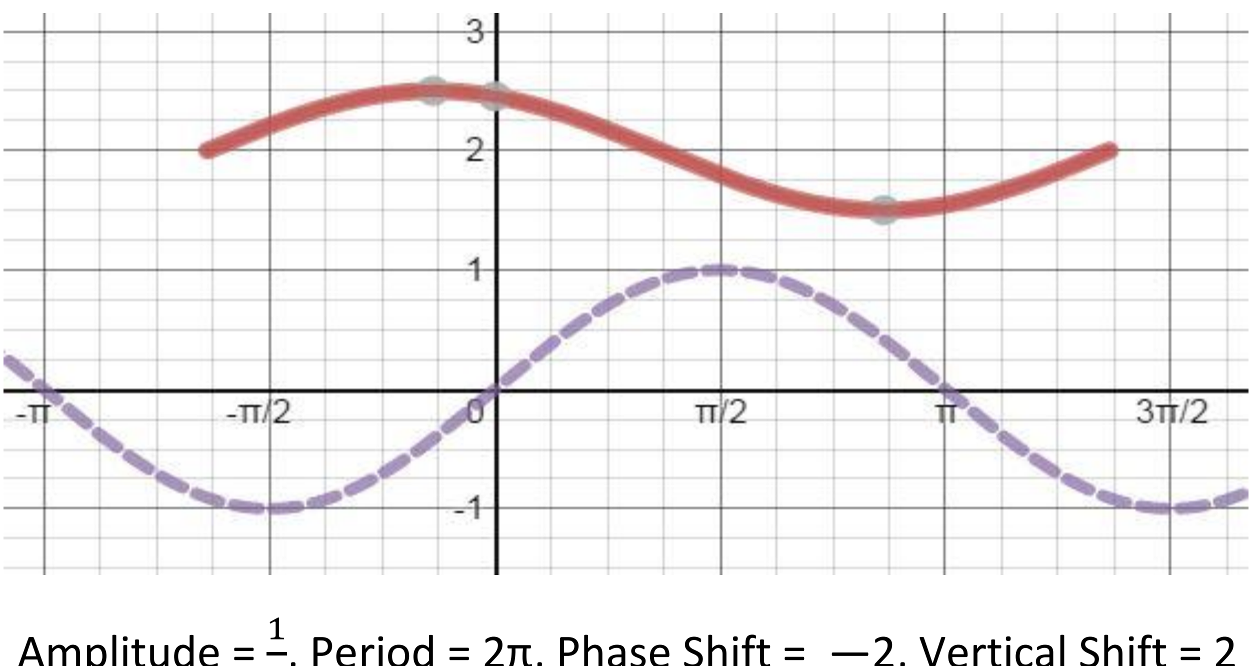
As the **phase shift** *increases*, the graph______________________, and as **phase shift** *decreases*, the graph___________________________

As the **vertical shift** *increases*, the graph__________________________, and as the **vertical shift** *decreases*, the graph__________________________ f(x) = 2sin(3x—3)+1

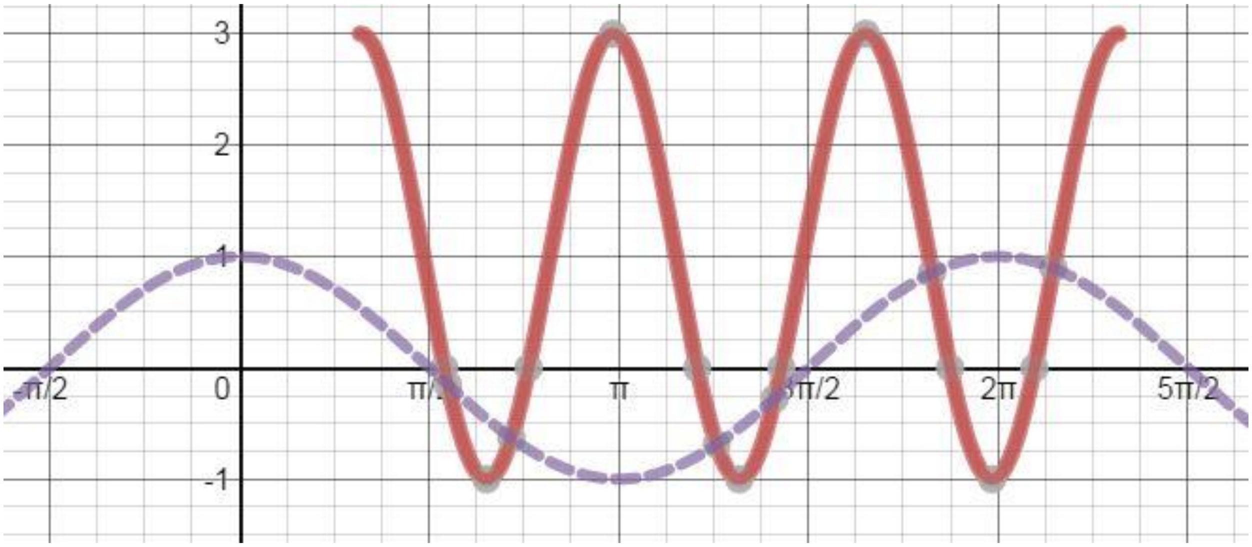


Amplitude = 2, Period = $\frac{2\pi}{3}$, Phase Shift = 1, Vertical Shift = 1

f(x) = $\frac{1}{2}$sin(x+2)+2

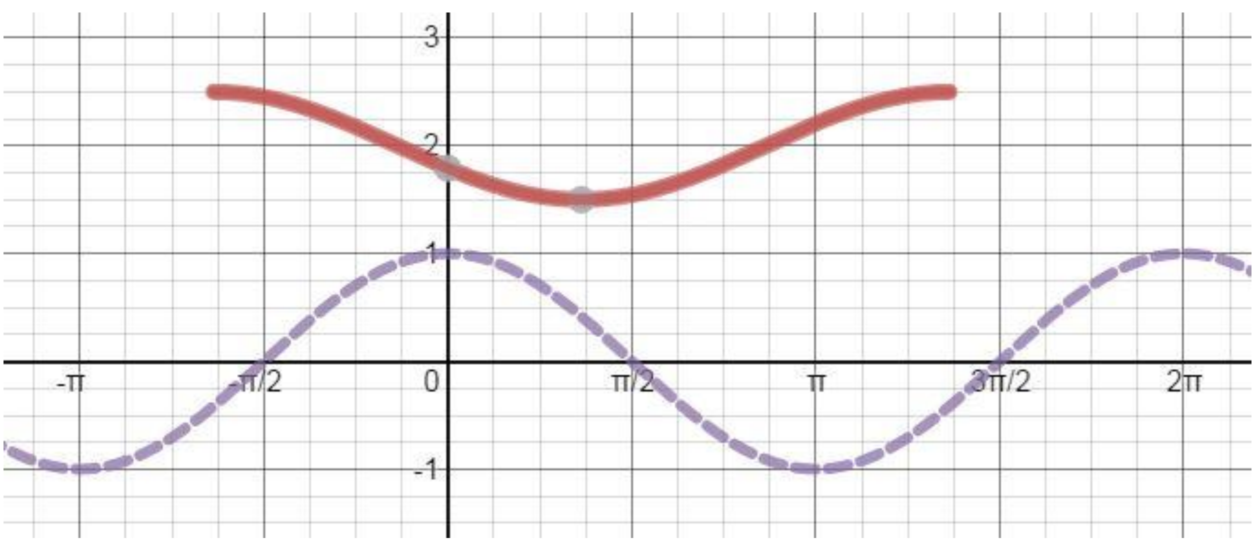


Amplitude = $\frac{1}{2}$, Period = 2π, Phase Shift = —2, Vertical Shift = 2 f(x) = 2cos(3x—3)+1



Amplitude = 2, Period = $\frac{2\pi}{3}$, Phase Shift = 1, Vertical Shift = 1

f(x) = $\frac{1}{2}$cos(x+2)+2

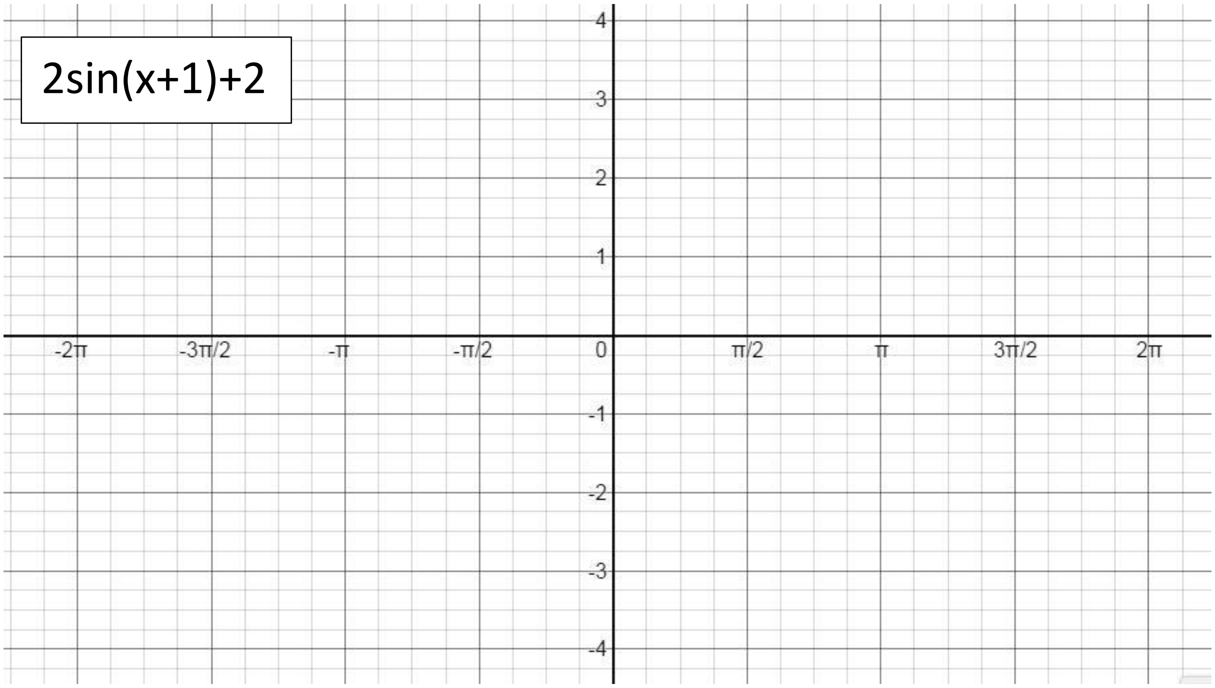


Amplitude = $\frac{1}{2}$, Period = 2π, Phase Shift = —2, Vertical Shift = 2

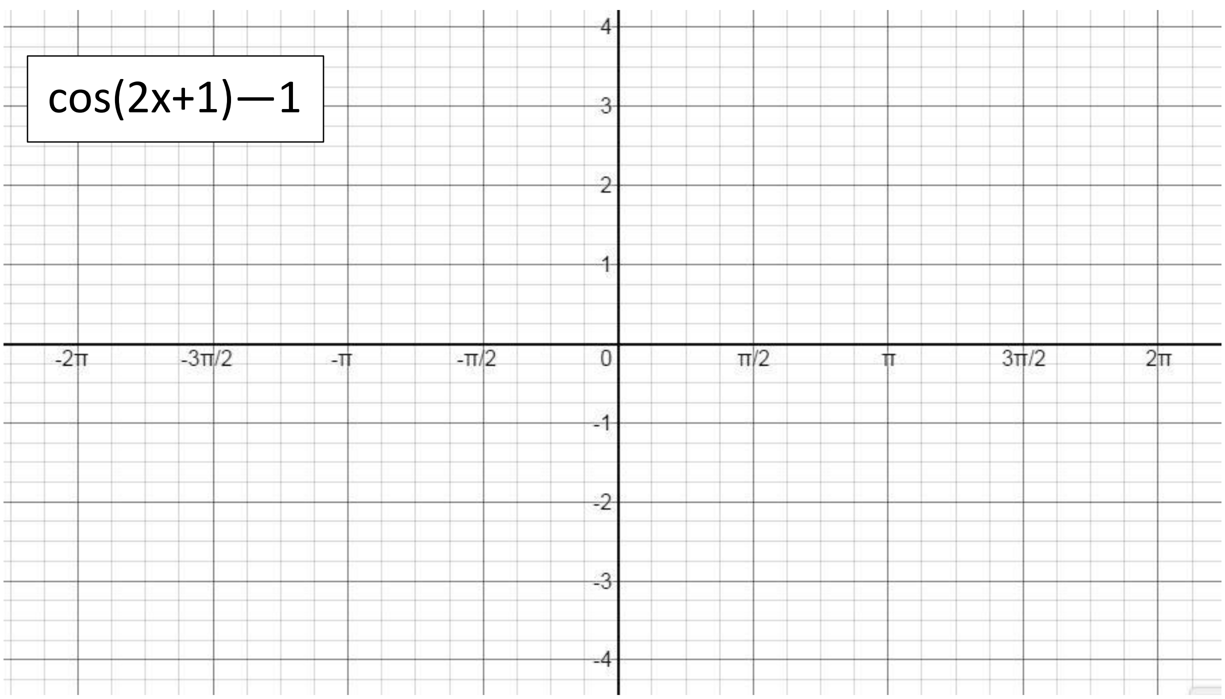
¡ sin(x) and cos(x) transformations !

Name:______________________________ Learning Group:_______

Graph the following functions and identify the amplitude, period, phase shift, and vertical shift



Amplitude =____ Period =______ Phase Shift =____ Vertical Shift =____

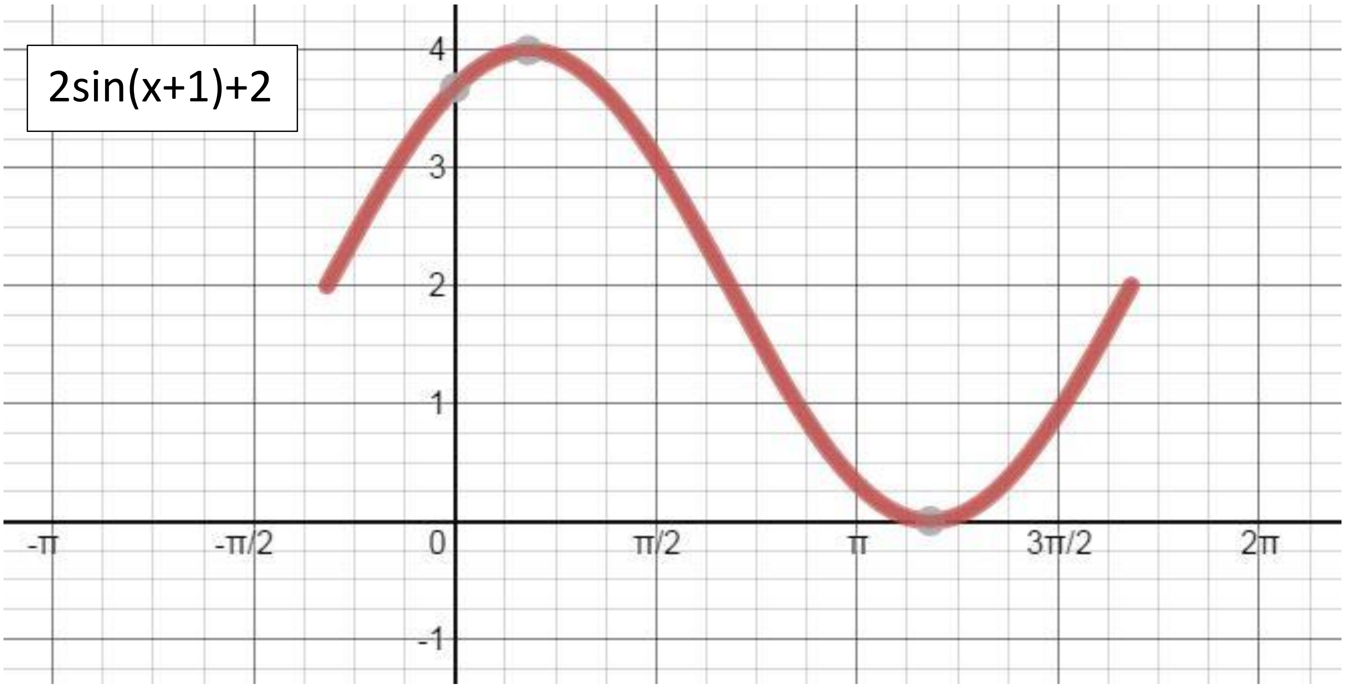


Amplitude =____ Period =______ Phase Shift =____ Vertical Shift =____

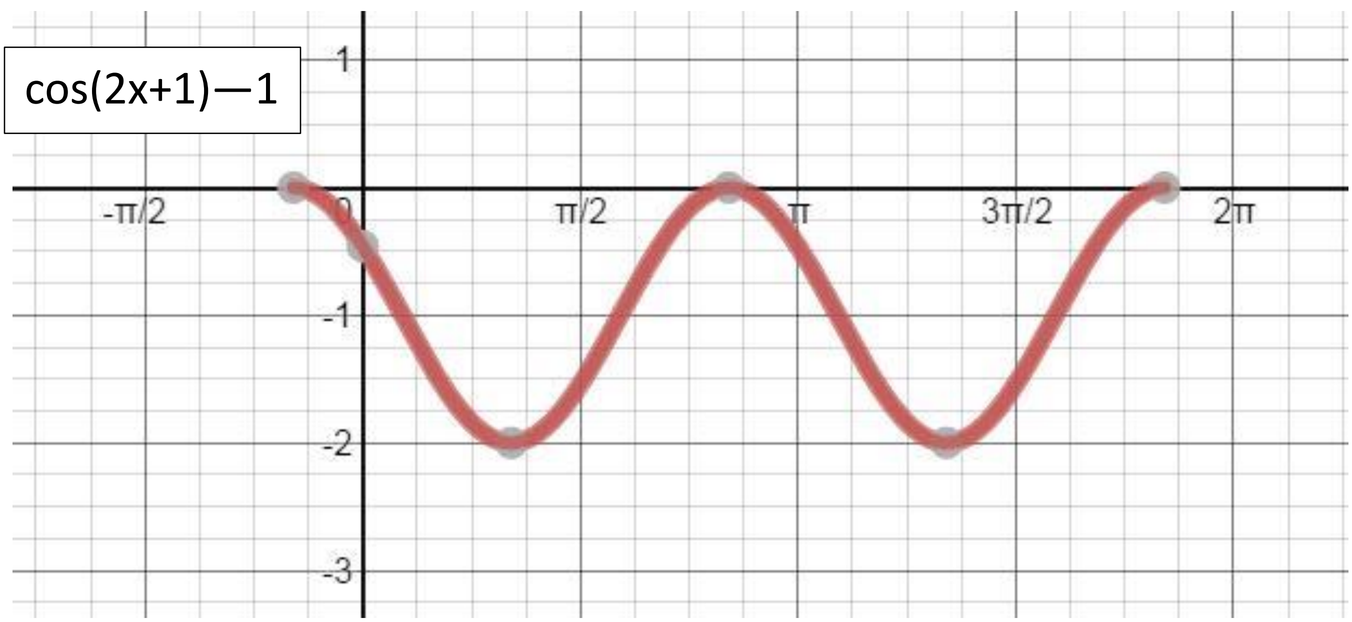
¡ sin(x) and cos(x) transformations !

Name:_____________________________ Learning Group:_______

Graph the following functions and identify the amplitude, period, phase shift, and vertical shift



Amplitude =_2__ Period =_2π__ Phase Shift =__-1__ Vertical Shift =_2__



Amplitude =_1_ Period =__π__ Phase Shift =$-\frac{1}{2}$ Vertical Shift =_-1_

Roller Coaster Extravaganza

Partner up!

<u>Start by using the sin(x) function, and asin(bx+c)+d to create your coaster</u>

1. Name your Roller Coaster!
2. Choose the height of your roller coaster. This will be what part of the sin(x) function?
    a. It should not go ‘below ground’
    b. So you’ll need to figure out the vertical shift as well
3. Choose how far you want your roller coaster to go
4. Choose how many cycles your roller coaster will have before it concludes the ride.
5. Find the period and the frequency
    a. $\frac{2\pi * cycles}{distance} = period$
    b. $\frac{1}{period} = frequency$
6. Write down your function thus far with a value for ‘c’ of $-\frac{\pi}{2}$:
    a. Phase shift = $\frac{-c}{b}$ don’t forget! Phase Shift =
7. Use the table on the back to make your points and graph the coaster!

$normal\ points$	frq	$points \times frequncy$	$phase\ shift$	$points + phase\ shift$	$y - values$

Function(Composition)

Function composition is applying one function to the result of another!

Let there be two functions f(x) and g(x)

(f∘g)(x)=_________, and we will take the_________function and plug it into wherever we see an________in

the_________function

Example:

$$f(x) = x^2 \text{ and } g(x) = x + 1$$

Find $f(g(x))$: We will take____________and plug it into wherever we see an______in the $f(x)$ function.

So $f\big(g(x)\big) = (\_\_\_\_\_\_\_\_\_\_)^2$

Does $f\big(g(x)\big)$ *always equal* $g\big(f(x)\big)$? ____________

$f\big(g(x)\big) = g(f(x))$ only when the two functions are:

____________________!

Inverse Functions

Given a function $y = 2x - 5$

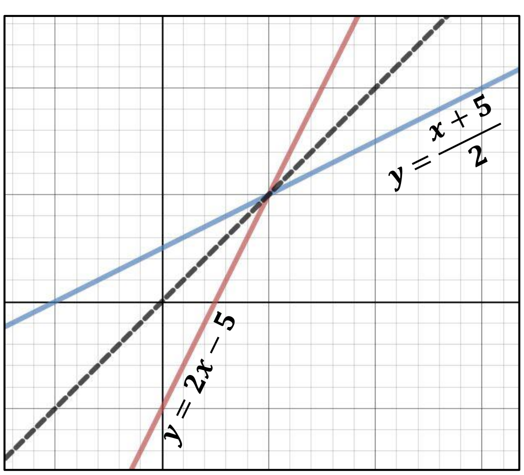
Take the function and ____________the two variables.

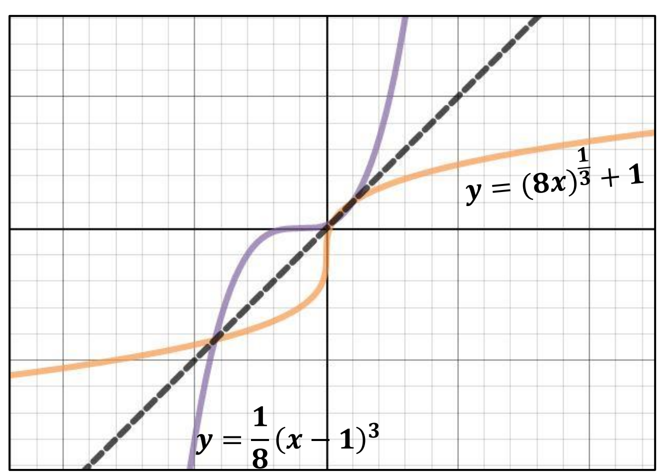
Then solve for___!

- So $y = 2x - 5$ would turn into__________________,
  and the inverse would equal____________________.

To check to see if the two functions are inverses, we can use function ______________________. If __________=___, and _________=___, we know the two functions are inverses.

- If we graph the two functions, they will reflect about the y=____ line.





RATIONAL *FUNCTION* BIBLE

A rational function is an expression that is the ratio of two polynomials.

There are three possibilities for our expression

$$\frac{f(x)}{g(x)}$$

_______ >______ | ______ =______ | ______ <______

1. First, we want to see if we can________________the expression. If we can get something to cancel in the ___________________, we know there will be a ___________________ ___________________at that point.

2. Second, we want to find____________ ______________.

To find these, we take the______________________and set it equal to _________.

3. Third, we find any other asymptotes by comparing the degrees of the ________________and ____________________

YAAAAAS

Removable Discontinuities

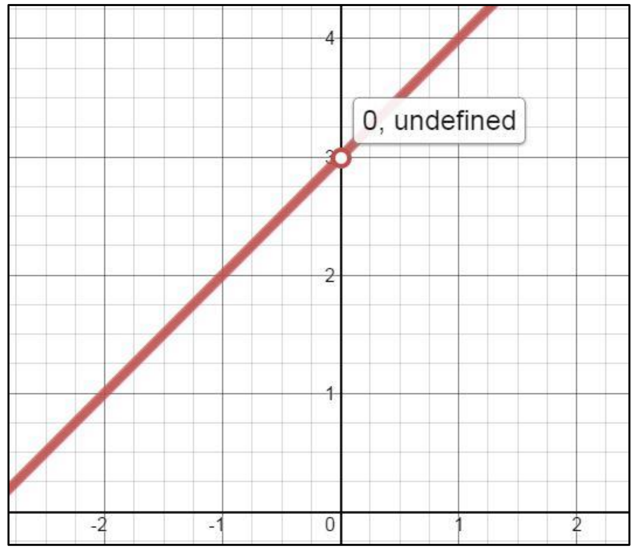
Example: $\frac{x^2+3x}{x}$

This simplifies to____________

and there is a ______________

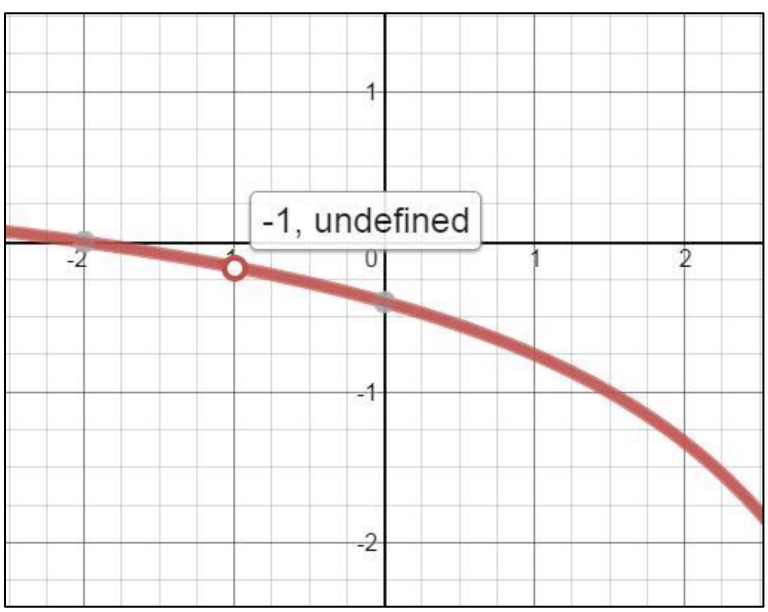
__________________ on the

graph at x = _____.



Example: $\frac{x^2+3x+2}{x^2-4x-5}$

This factors to $\frac{(\qquad)(\qquad)}{(\qquad)(\qquad)}$ and there is a removable discontinuity at the point x =_______.



For $f(x) = g(x)$

- This means the______________in the numerator is the same as the_____________in the denominator!

So we will take the_______________ ___________in the numerator and put it over the_____________ ___________in the denominator.

- After simplifying, the result will represent a

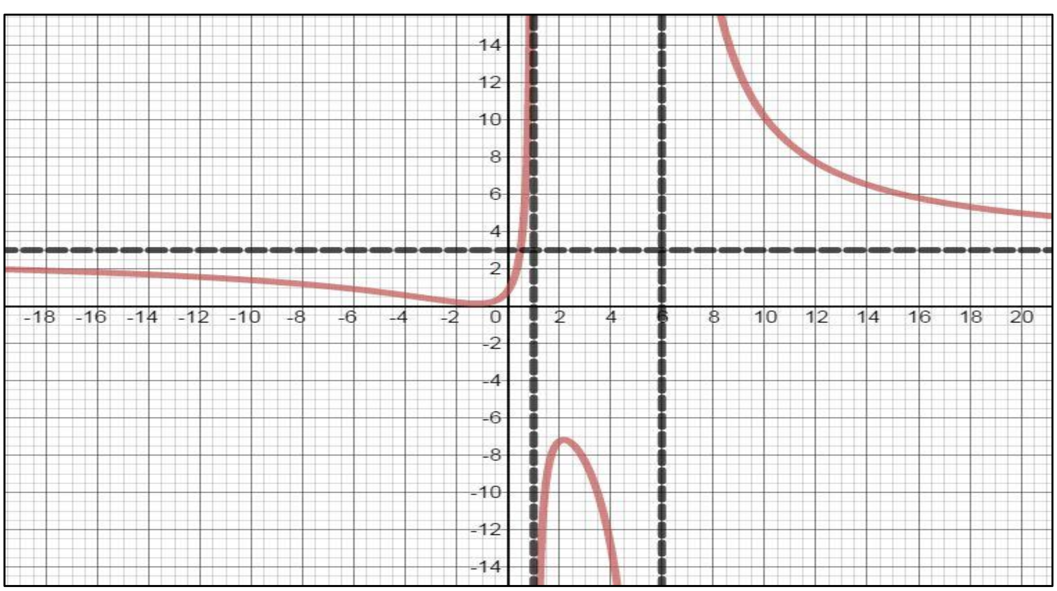
  _______________ ___________________on the graph.

Example: $\frac{3x^2+6x+5}{x^2-7x+6}$

Step 1. Does anything cancel?________

Step 2. Are there vertical asymptotes?________
Where?_________________

Step 3. Are there horizontal asymptotes? _________
Where?_________________



For $f(x) < g(x)$

This could be considered

bottom heavy

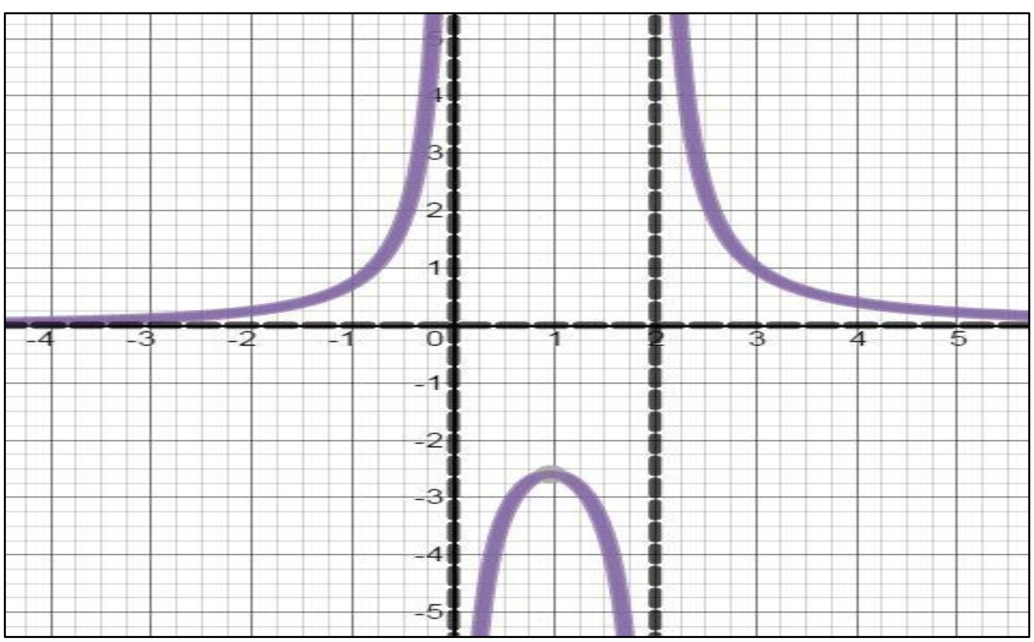
- This means the______________in the numerator is ____________than the____________in the denominator!
- This means there will ***ALWAYS*** be a ____________________ ____________________at______.

Example: $$\frac{x+12}{5x^2-10x}$$

Step 1. Does anything cancel?__________

Step 2. Are there any vertical asymptotes?_________
Where?________________

Step 3. Are there horizontal asymptotes?__________
Where?____________



For $f(x) > g(x)$

This could be considered

top heavy

- This means the_______________in the numerator is ________________than the_____________in the denominator!

After simplifying we must do__________ ____________to find out our _____________ ____________________!

Example: $\frac{x^2 - 3x}{x - 4}$

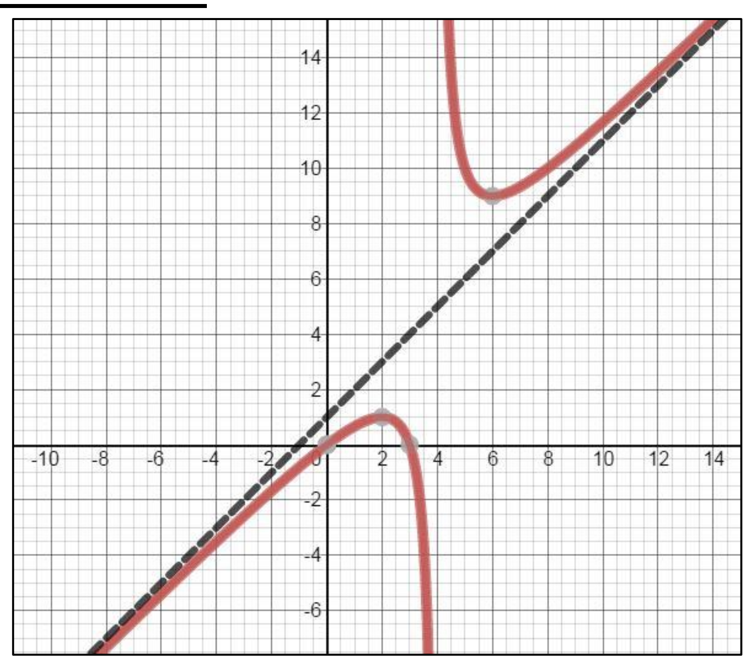
Step 1. Does anything cancel?__________

Step 2. Are there any vertical asymptotes?_________

Where?________________

Step 3. Is there a slant asymptote?_________

Where?____________________



End Behavior =______________

FINAL CELEBRATION OF KNOWLEDGE

Name:____________________________ Learning Group:_______

Find the inverse of the following functions:

1. $y = 2x + 4$

2. $y = 3x^2 - 5$

Evaluate the following function composition:

3. $f(x) = 2x$
   $g(x) = x + 3$

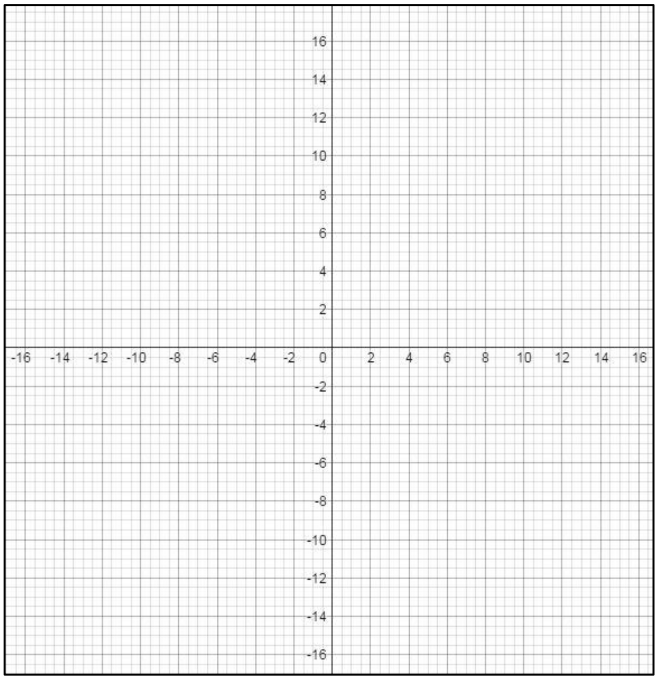
   $f(g(x)) =$

4. $f(x) = x^2 + 3x$
   $g(x) = x - 1$

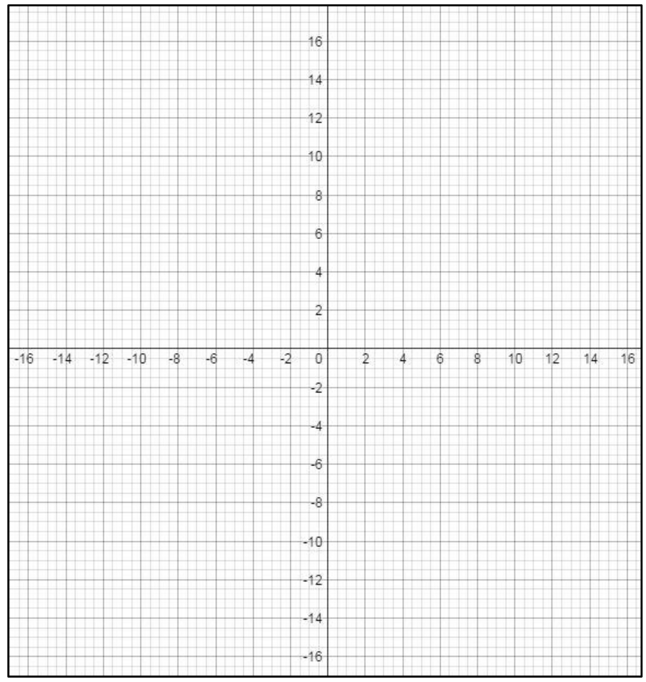
   $f(g(x)) =$

Graph a quick sketch of the following functions:

5. $f(x) = (x+3)^2 - 1$



6. $g(x) = x^3 - 5$



Identify **ALL** asymptotes (vertical, horizontal, or slant), removable discontinuities, and state the domain and range:

7. $\frac{x^2+6x+8}{x^2+x-12}$

8. $\frac{x+1}{3x-4}$

9. $\frac{x-2}{x^2-5x-6}$

10. $\frac{2x^2+4x+5}{x-3}$

Identify the amplitude, period, frequency, phase shift, and vertical shift of each of the following functions:

11. $3\sin(2x - 4) + 5$

Period:

Amplitude:

Vertical Shift:

Phase Shift:

Frequency:

12. $-2\cos\left(3x + \frac{\pi}{2}\right) - 1$

Period:

Amplitude:

Vertical Shift:

Phase Shift:

Frequency:

13. Fill in the first quadrant of the unit circle with all the radians, degrees, and points. If you can do more feel free!